SIGNALS AND SYSTEMS
For
Graduate Aptitude Test in Engineering

By

Dr. N. Balaji

Professor of ECE

JNTUK

Kakinada

Session: 1

Topic : Introduction and Basics of Signals and Systems

Date : 11.05.2020

By

Dr. N. Balaji

Professor of ECE

JNTUK

Kakinada

Syllabus

- **Continuous-time signals:** Fourier series and Fourier transform representations, sampling theorem and applications; Discrete-time signals: discrete-time Fourier transform (DTFT), DFT, FFT, Z-transform, interpolation of discrete-time signals;

- **LTI systems:** definition and properties, causality, stability, impulse response, convolution, poles and zeros, parallel and cascade structure, frequency response, group delay, phase delay, digital filter design techniques.

Outline of the Session

- **Introduction to Signals**
- **Classification of Signals**
- **Classification of Systems**
- **Signal operations**
- **GATE previous questions**

Analysis of weightage given In GATE Papers (Signals & Systems)

Analysis of weightage given In GATE Papers (Signals & Systems)

Year	ECE
2020	10
2019	12
2018	9
2017	11
Over All average Marks	10.50

Introduction to Signals

- A Signal is a function of one or more independent variables that conveys information to represent a physical phenomenon.
- For a function f, in the expression $f(t_1, t_2, \cdots, t_n)$, each of the $\{t_k\}$ is called an independent variable, while the function value itself is referred to as a dependent variable.
- Some examples of signals include:
  - A voltage or current in an electronic circuit
  - the position, velocity, or acceleration of an object
  - a force or torque in a mechanical system
  - a flow rate of a liquid or gas in a chemical process
  - a digital image, digital video, or digital audio

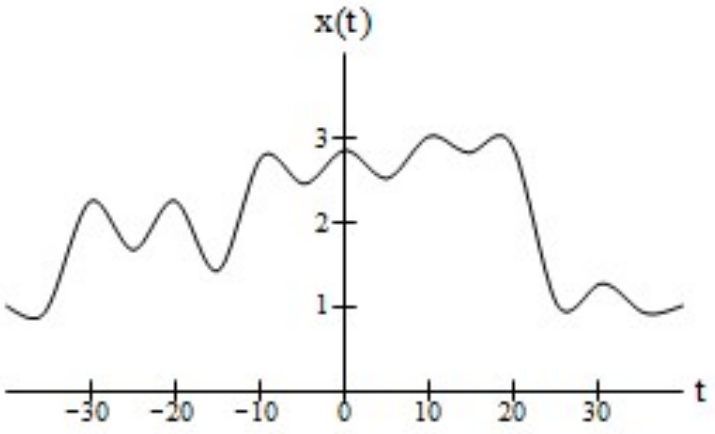
Acknowledgement : Lecture slides of Michael D. Adams

Classification of Signals

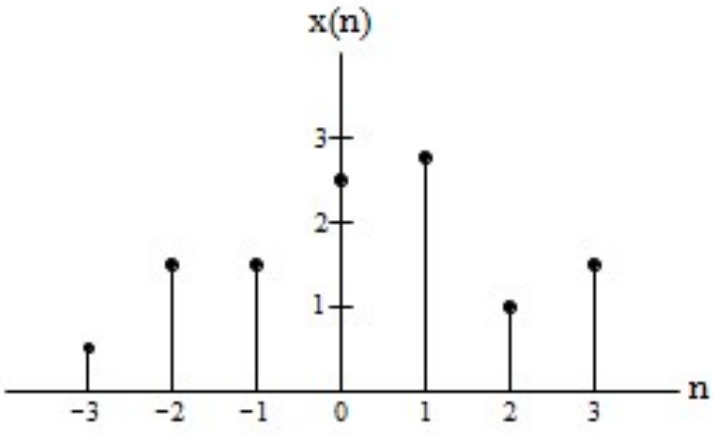
- Number of independent variables (i.e., dimensionality):
  - A signal with **one** independent variable is said to be one dimensional (e.g., audio).
  - A signal with **more than one** independent variable is said to be **multi-dimensional** (e.g., image).
- Continuous or discrete independent variables:
  - A signal with **continuous** independent variables is said to be **continuous time** (CT) (e.g., voltage waveform).
  - A signal with **discrete** independent variables is said to be **discrete time** (DT) (e.g., stock market index).
- Continuous or discrete dependent variable:
  - A signal with a **continuous** dependent variable is said to be **continuous valued** (e.g., voltage waveform).
  - A signal with a **discrete** dependent variable is said to be **discrete valued** (e.g., digital image).
- A continuous-valued CT signal is said to be analog (e.g., voltage waveform).
- A discrete-valued DT signal is said to be digital (e.g., digital audio).

Acknowledgement : Lecture slides of Michael D. Adams

Graphical Representation of Signals



Continuous-Time (CT) Signal

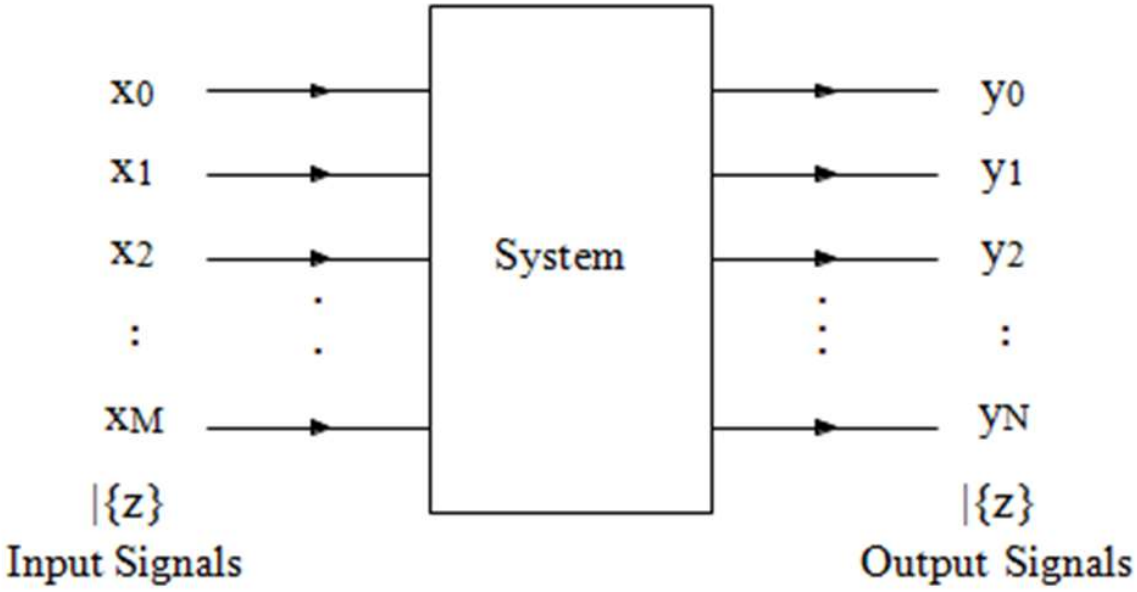


Discrete-Time (DT) Signal

Acknowledgemet : Lecture slides of Michael D. Adams

System

- A system is an entity that processes one or more input signals in order to produce one or more output signals.



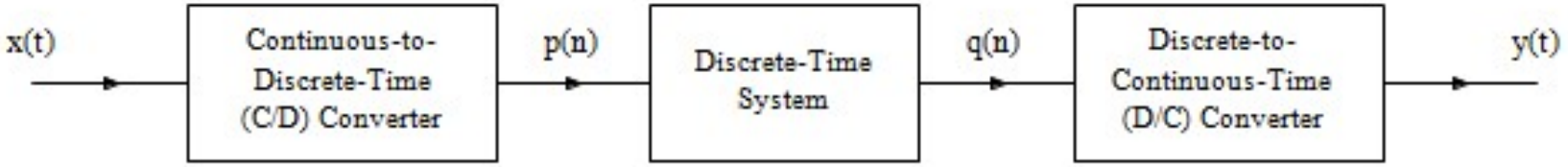
Acknowledgemet : Lecture slides of Michael D. Adams

Classification of Systems

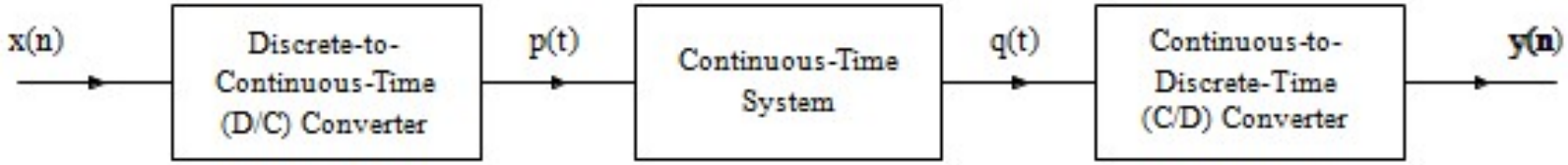
- Number of Inputs
  - A system with **one** input is said to be **single input (SI)**
  - A system with **more than one** input is said to be **multiple input (MI)**
- Number of Outputs
  - A system with **one** output is said to be **single output (SO)**
  - A system with **more than one** input is said to be **multiple output (MO)**
- Types of signals processed:
  - A system can be classified in terms of the **type of signals** that it processes.
  - Consequently, terms such as the following (which describe signals) can also be used to describe systems:
    - One-dimensional and multi-dimensional
    - Continuous-time (CT) and discrete-time (DT)
    - Analog and digital
  - For example, a continuous-time (CT) system processes CT signals and a discrete-time(DT) system processes DT signals

Acknowledgemet : Lecture slides of Michael D. Adams

Signal Processing Systems



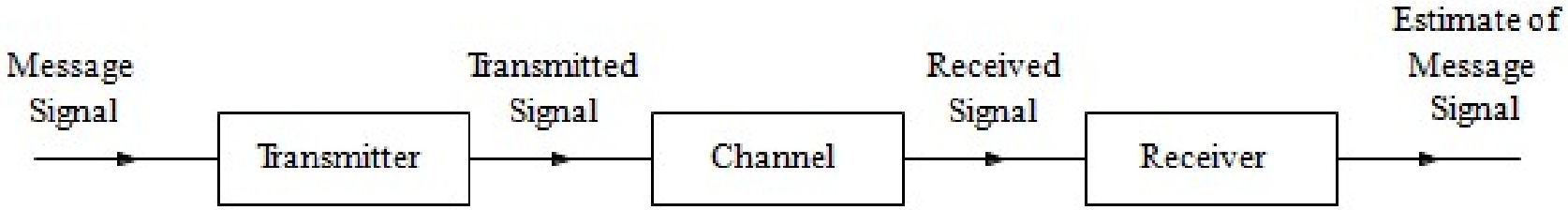
Processing a Continuous-Time Signal With a Discrete-Time System



Processing a Discrete-Time Signal With a Continuous-Time System

Acknowledgemet : Lecture slides of Michael D. Adams

Communication Systems



General Structure of a Communication System

Acknowledgemet : Lecture slides of Michael D. Adams

Different operations on Signal

- Time shifting
- Amplitude shifting
- Amplitude Scaling
- Time scaling
- Time reversal
- Amplitude reversal

Acknowledgemet : Lecture slides of Michael D. Adams

Time Shifting(Translation)

- **Time shifting is, the shifting of a signal in time.**
- **This is done by adding or subtracting the amount of the shift to the time variable in the function.**
- **When t ≥0,**
  i. Subtracting a fixed amount from the time variable will shift the signal to the right (delay) that amount
  ii. while adding to the time variable will shift the signal to the left (advance).

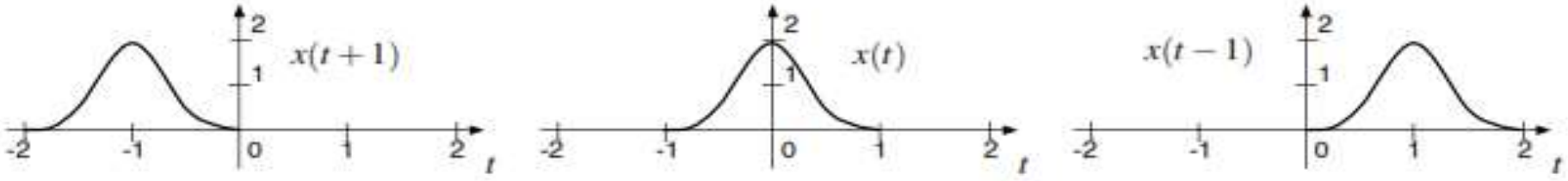
Acknowledgemet : Lecture slides of Michael D. Adams and Prof. Paul Cuff

Time Shifting

Time Shift

For a continuous-time signal $x(t)$, and a time $t_1 > 0$,

- Replacing t with $t - t_1$ gives a *delayed* signal $x(t - t_1)$
- Replacing t with $t + t_1$ gives an *advanced* signal $x(t + t_1)$

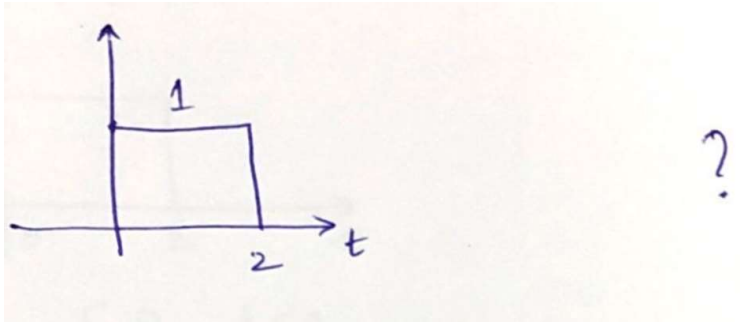


Acknowledgemet : Lecture slides of Michael D. Adams and Prof. Paul Cuff

Time Shifting(Translation)

- Time Shifting:

$$x(t) \longrightarrow y(t) = x(t+k)$$

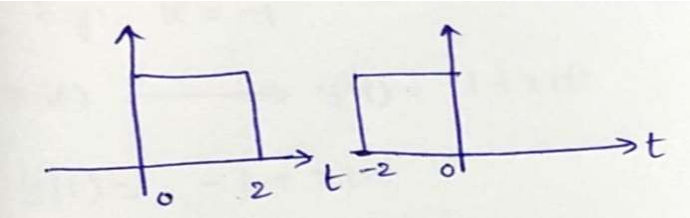


Acknowledgemet : Lecture slides of Michael D. Adams

Left shift and Right Shift

- Case (i): when $k > 0$

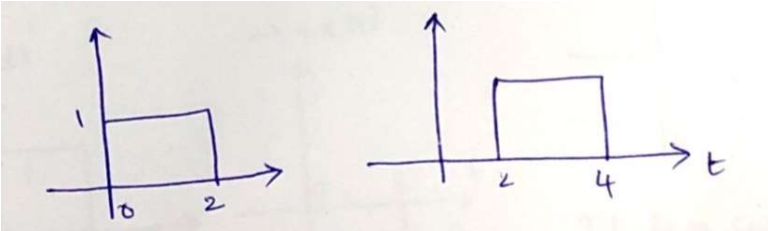
$$x(t) \longrightarrow y(t) = x(t+2)$$



It is the case of left shifting

- Case (ii): when $k < 0$

$$x(t) \longrightarrow y(t) = x(t-2)$$

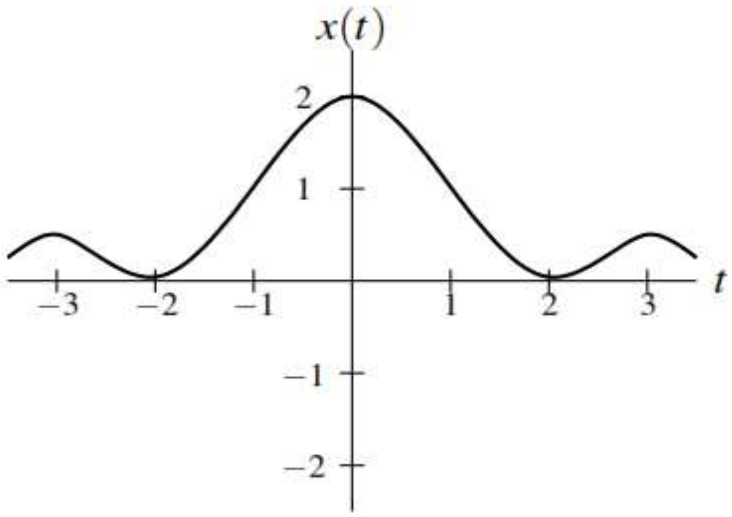


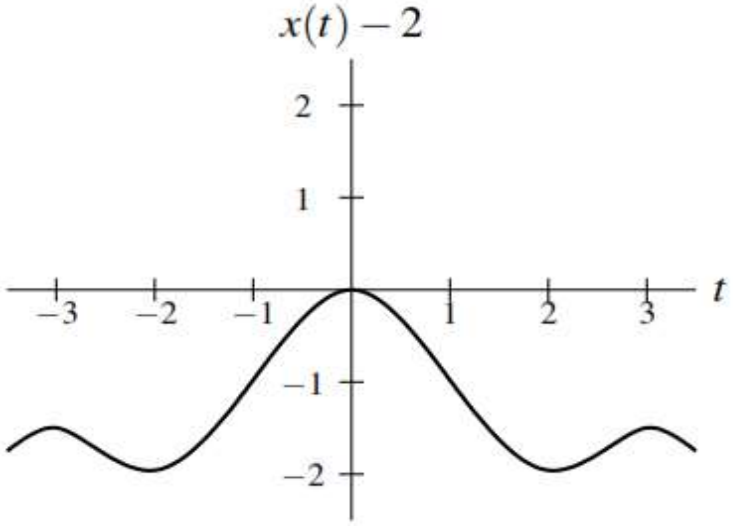
It is the case of right shifting

Acknowledgement : Lecture slides of Michael D. Adams

Amplitude Shifting

- Amplitude shifting maps the input signal x to the output signal y as given by $y(t) = x(t) + b$, where b is a real number.
- Geometrically, amplitude shifting adds a vertical displacement to x

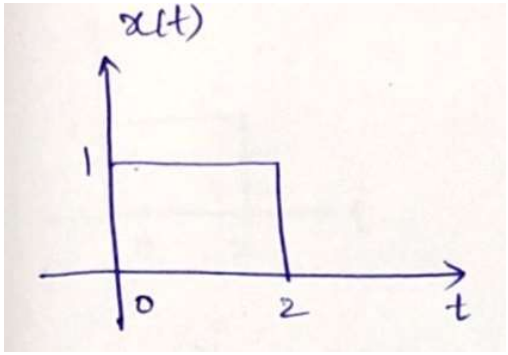




Acknowledgement : Lecture slides of Michael D. Adams

Amplitude Shifting

$$x(t) \longrightarrow y(t) = x(t) + k$$



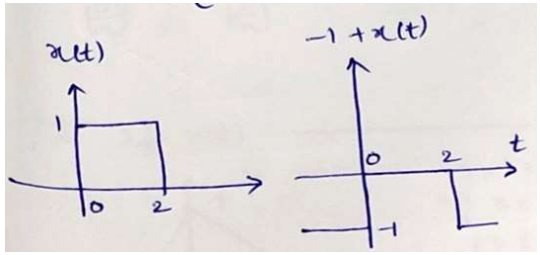
$$x(t) = \begin{cases} 0 & t < 0 \\ 1 & 0 < t < 2 \end{cases}$$

- Case (i): when $k < 0$

$$x(t) \longrightarrow y(t) = -1 + x(t)$$

$$y(t) = \begin{cases} -1+0 & t < 0 \\ -1+1 & 0 \le t \le 2 \\ -1+0 & t > 2 \end{cases}$$

$$= \begin{cases} -1 & t < 0 \\ 0 & 0 \le t \le 2 \\ -1 & t > 2 \end{cases}$$



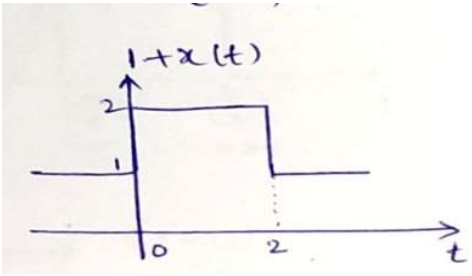
Its case of downward shifting

- Case (ii): when $k > 0$

$$x(t) \longrightarrow y(t) = 1 + x(t)$$

$$y(t) = \begin{cases} 1+0 & t < 0 \\ 1+1 & 0 \le t \le 2 \\ 1+0 & t > 2 \end{cases}$$

$$= \begin{cases} 1 & t < 0 \\ 2 & 0 \le t \le 2 \\ 1 & t > 2 \end{cases}$$

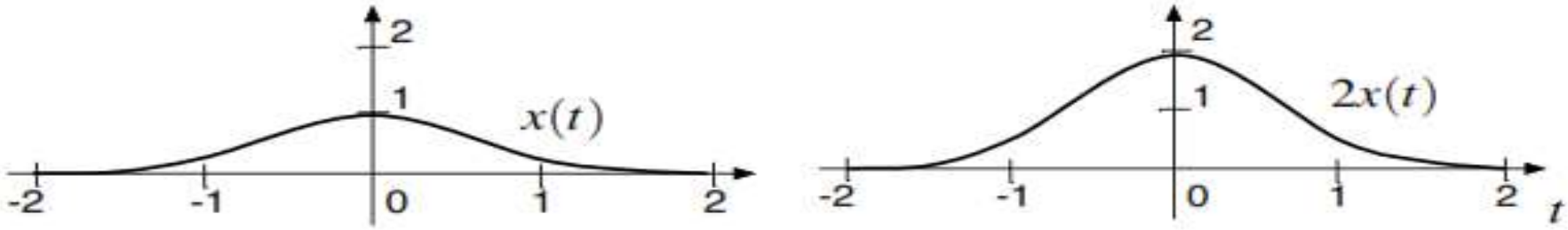


Its case of upward shifting

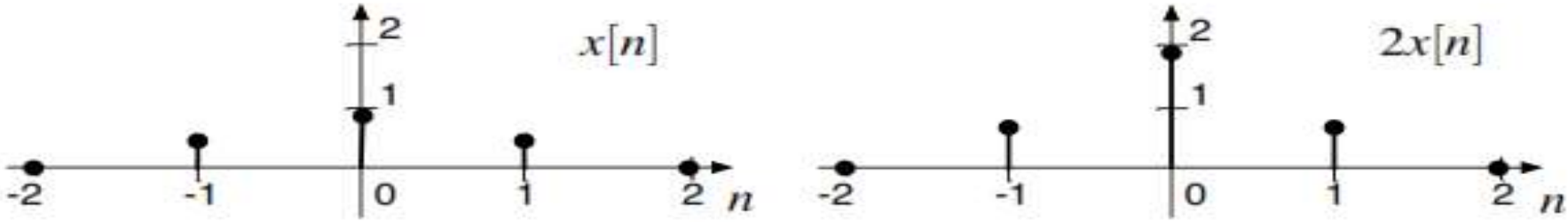
Acknowledgement : Lecture slides of Michael D. Adams

Amplitude Scaling

- The scaled signal $ax(t)$ is $x(t)$ multiplied by the constant a



- The scaled signal $ax[n]$ is $x[n]$ multiplied by the constant a



Acknowledgement : Lecture slides of Michael D. Adams

Time Scaling

- Time scaling maps the input signal x to the output signal y as given by by
  $y(t) = x(at)$, where a is a nonzero real number.
- Such a transformation is associated with a dilation (i.e., compression/expansion along the time axis) and/or time reversal.
- If |a| > 1, the signal is compressed along the time axis by a factor of |a|.
- If |a| < 1, the signal is expanded (i.e., stretched) along the time axis by a factor of $\left|\frac{1}{a}\right|$

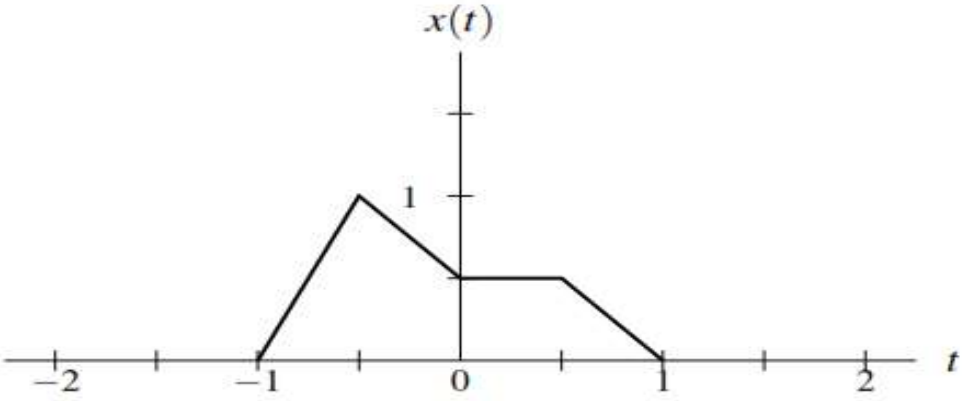
Acknowledgement : Lecture slides of Michael D. Adams

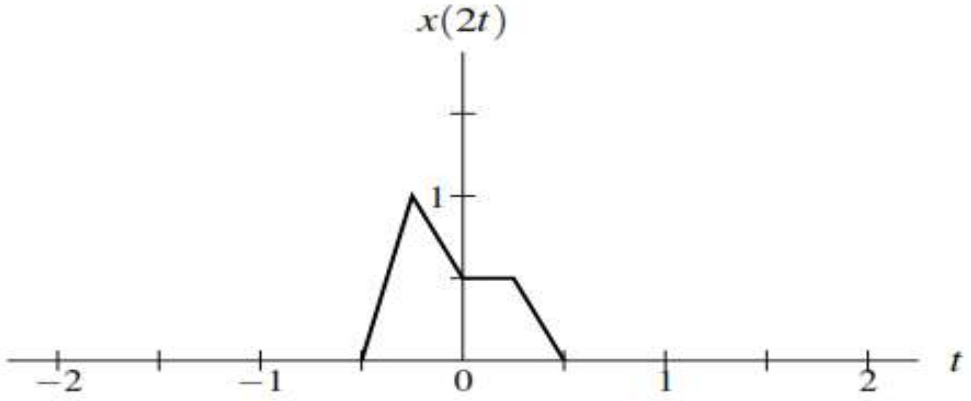
Time Scaling

- If $|a|=1$, the signal is neither expanded nor compressed.
- If $a < 0$, the signal is also reversed.
- Dilation (i.e., expansion/compression) and time reversal **commute.**
- Time reversal is a special case of time scaling with $a = -1$; and time compression/ expansion is a special case of time scaling with $a > 0$

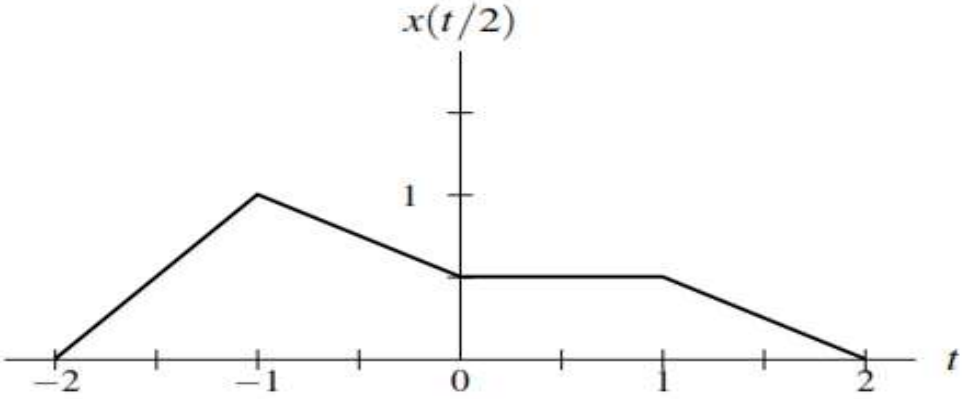
Acknowledgement : Lecture slides of Michael D. Adams

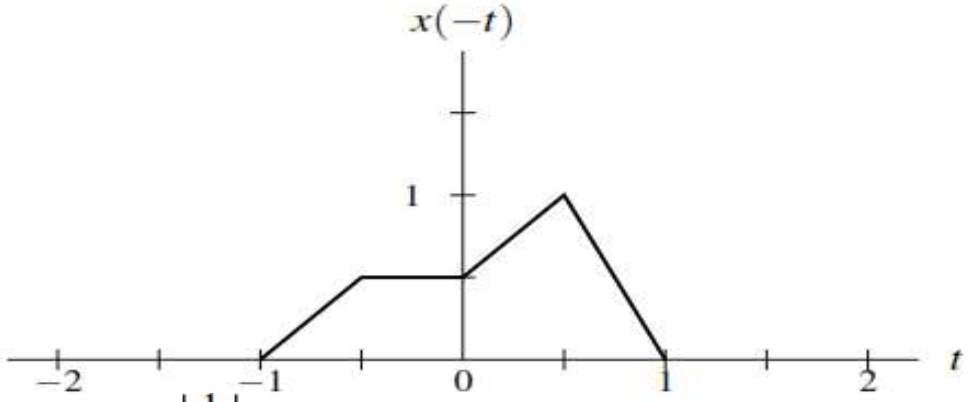
Time Scaling





If |a| > 1, the signal is compressed along the time axis by a factor of |a|





If |a| < 1, the signal is expanded (i.e., stretched) along the time axis by a factor of $\left|\frac{1}{a}\right|$

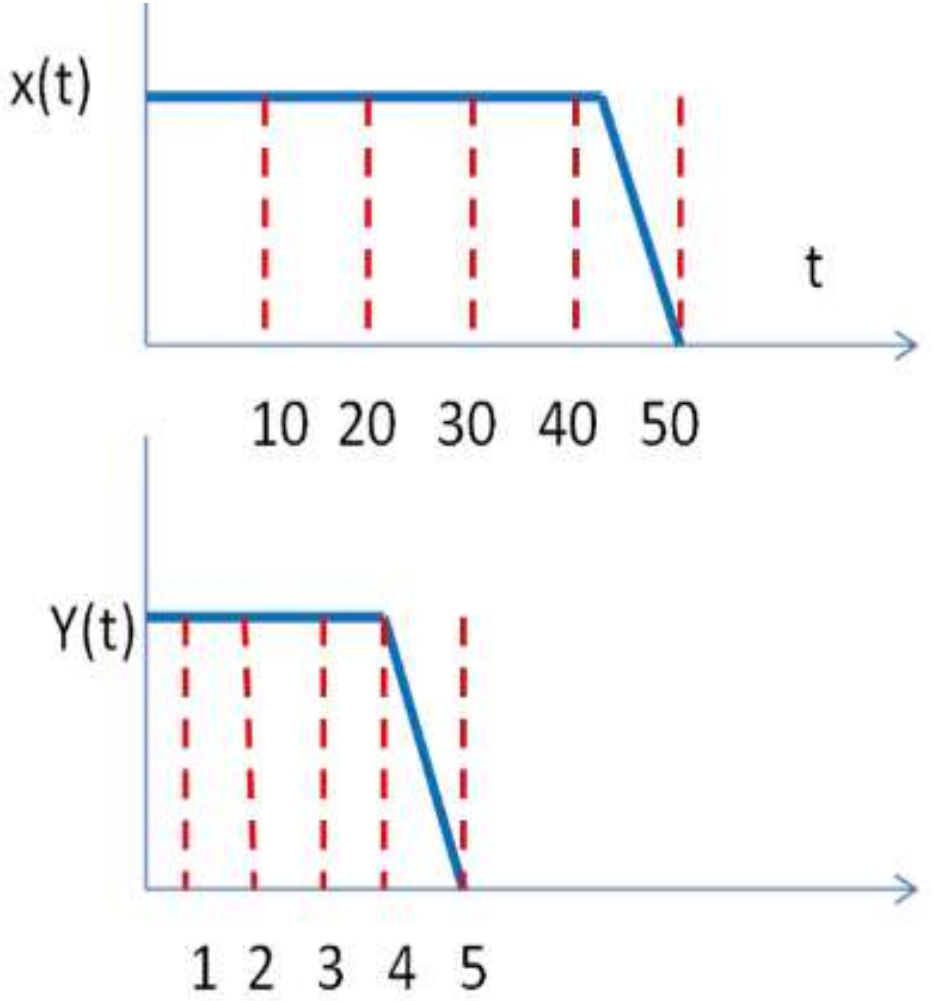
Time Scaling

Consider an example y(t) = x(10t)

For t=0, y(0) = x(0), t=1, y(1) = x(10), t=2, y(2) = x(20) …

…t=5, y(5) = x(50)

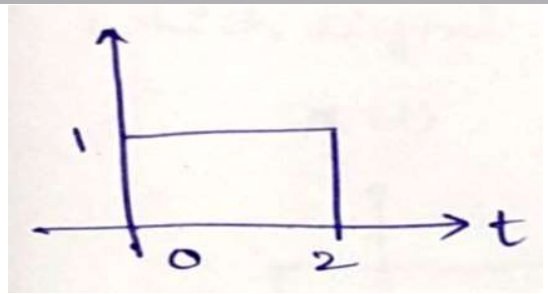
so, by the time y(t) goes to 5, x(t) takes values at t=50



Acknowledgement : Lecture slides of Michael D. Adams

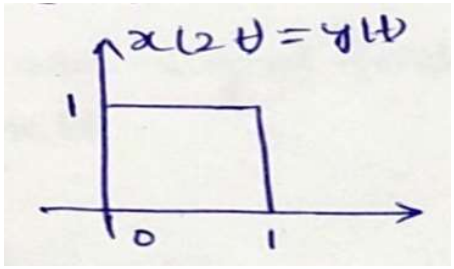
Time Scaling

$x(t) \longrightarrow y(t) = x(at)$



- Case (i): when $a > 1$

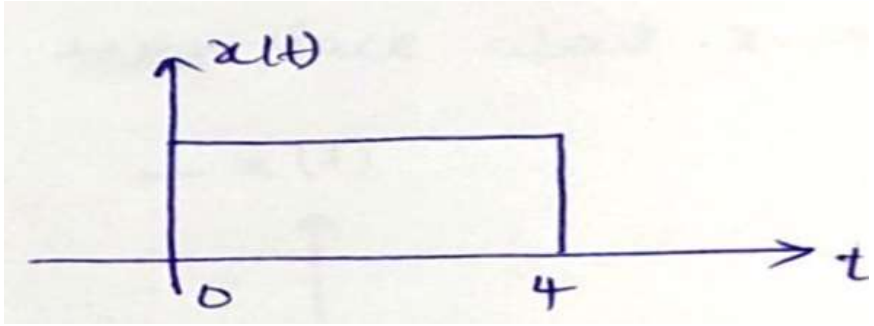
$x(t) \longrightarrow y(t) = x(2t)$



Time compression

- Case (ii): when $a < 1$

$x(t) \longrightarrow y(t) = x(0.5t)$



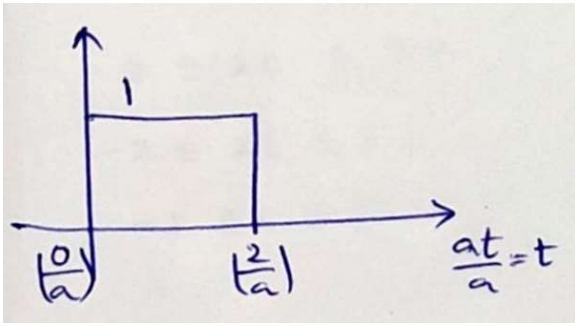
Time Expansion

Acknowledgemet : Lecture slides of Michael D. Adams

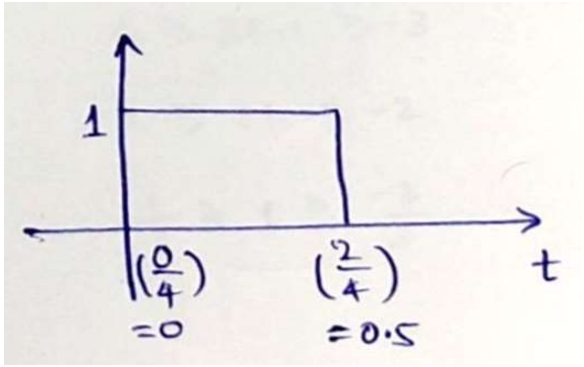
Time Scaling

Note: **General rule**

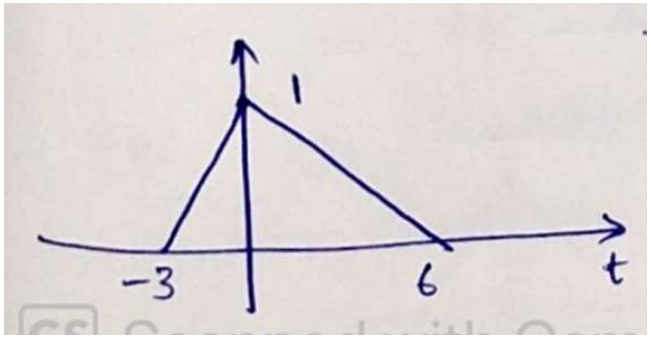
$x(t)$ ⟶ t



$x(4t)$



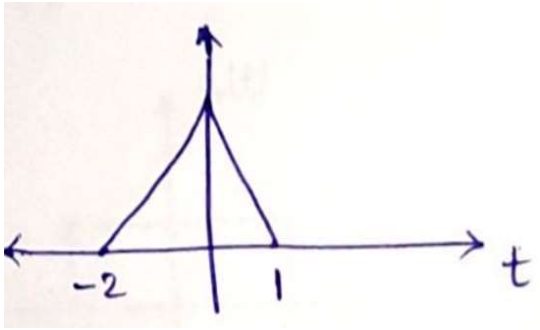
If $x(t)$ ⟶ $(x(-3t)$



$$-3 \le -3t \le 6$$

$$\frac{-3}{-3} \ge t \ge \frac{6}{-3}$$

$$1 \ge t \ge -2$$



Acknowledgemet : Lecture slides of Michael D. Adams

Time Reversal

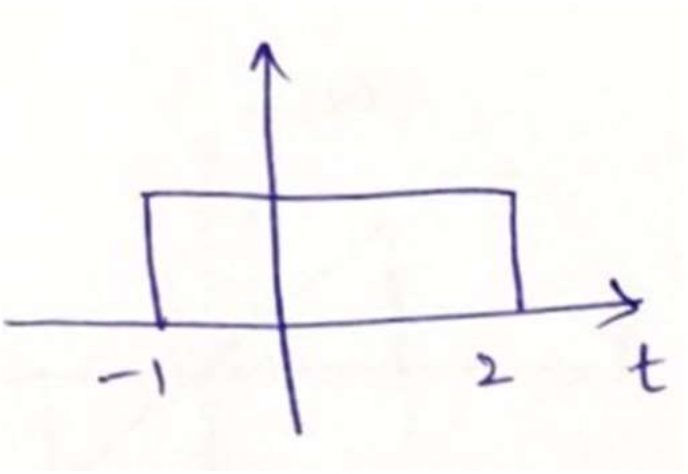
- If $a < 0$, the signal is also reversed.
- Dilation (i.e., expansion/compression) and time reversal **commute.**
- Time reversal is a special case of time scaling with $a = -1$; and time compression/ expansion is a special case of time scaling with $a > 0$

Acknowledgemet : Lecture slides of Michael D. Adams

Time Reversal

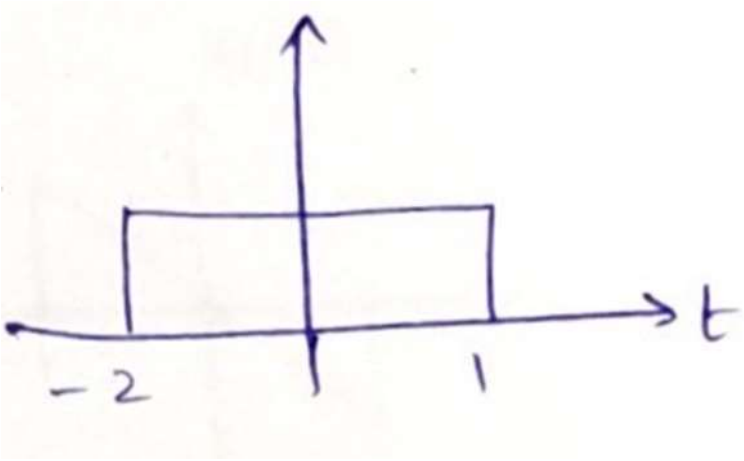
- Time reversal is a special case of time scaling in which signal folding will take place around y axis

$x(t)$ ⟶ $x(-t)$



$-1 \leq -t \leq 2$

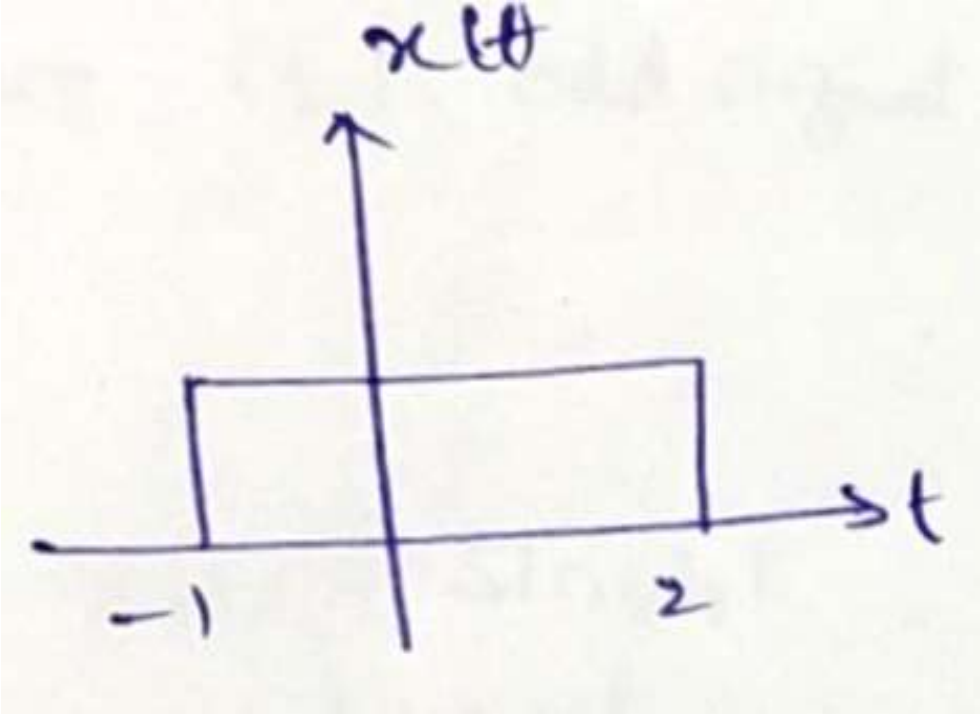
$1 \geq t \geq -2$

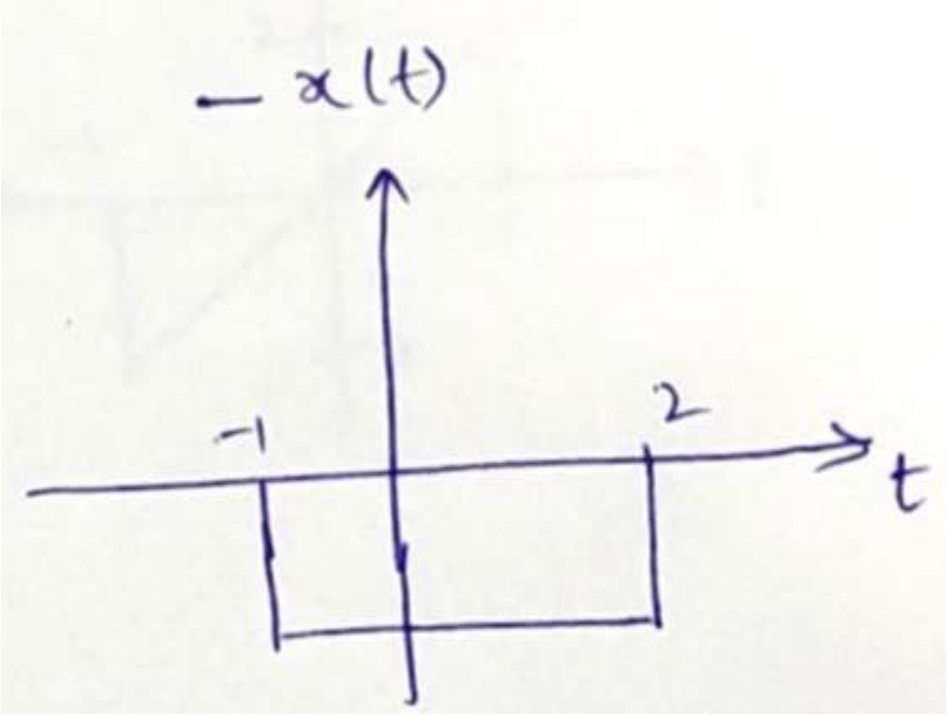


Acknowledgemet : Lecture slides of Michael D. Adams

Amplitude Reversal

- In this case, signal folding will take place about *x*-axis.





Acknowledgemet : Lecture slides of Michael D. Adams

Question1

For $x(t)$ indicated in Figure P1.6, sketch the following:

(a) $x(-t)$

(b) $x(t + 2)$

(c) $x(2t + 2)$

(d) $x(1 - 3t)$

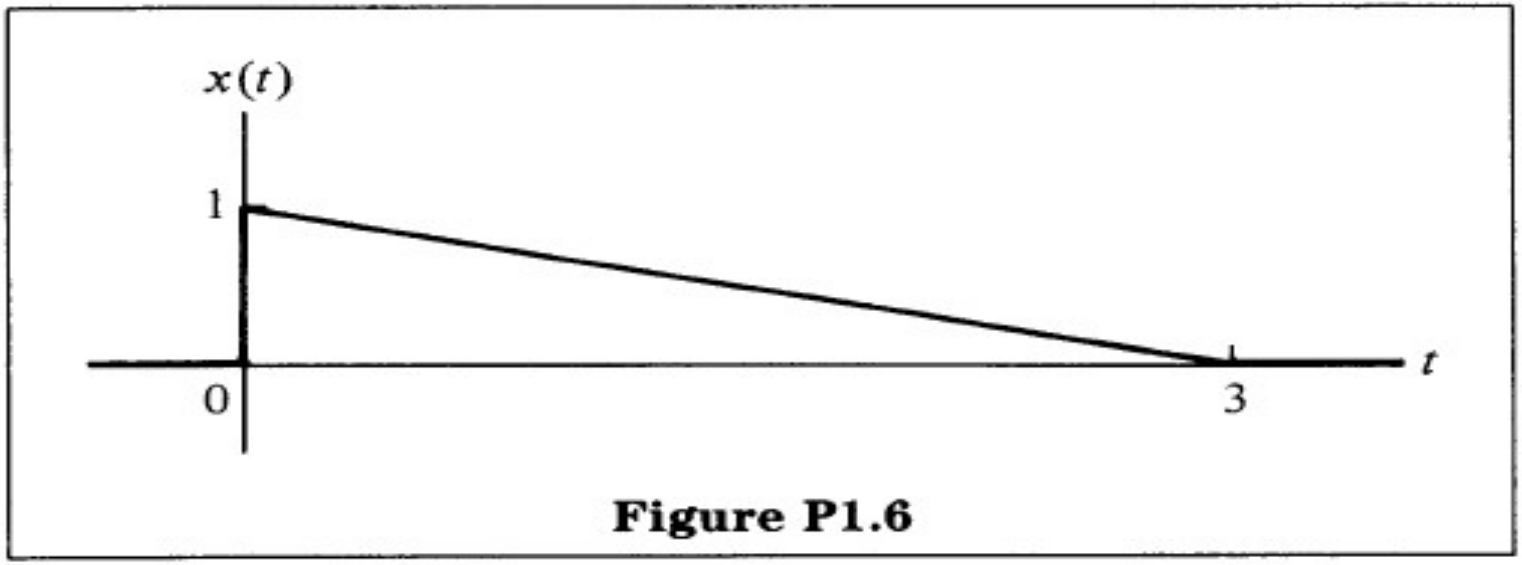
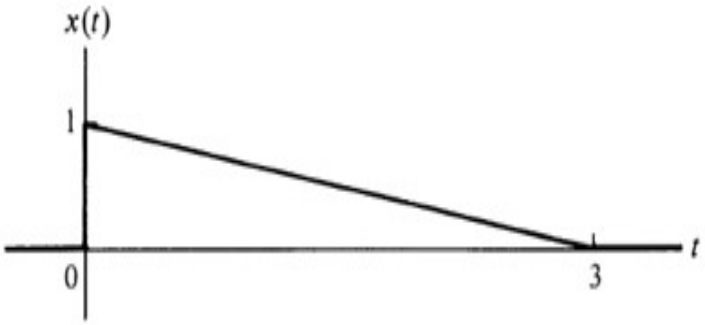


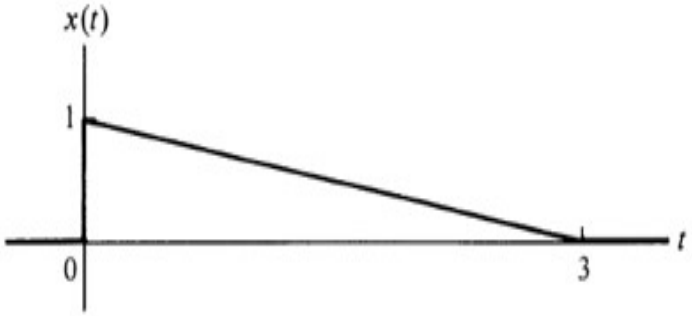
Figure P1.6

Acknowledgemet : Lecture slides of Michael D. Adams

Acknowledgement: OCW MIT Resource: Signals and Systems Professor Alan V. Oppenheim

Solution1





(a) This is just a time reversal.

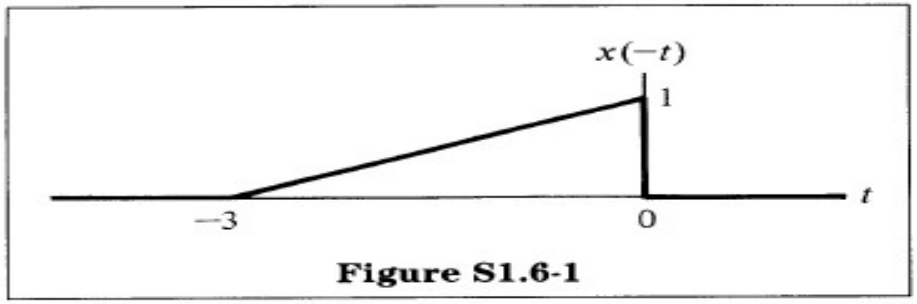


Figure S1.6-1

Note: Amplitude remains the same. Also, reversal occurs about $t = 0$.

(b) This is a shift in time. At $t = -2$, the vertical portion occurs.

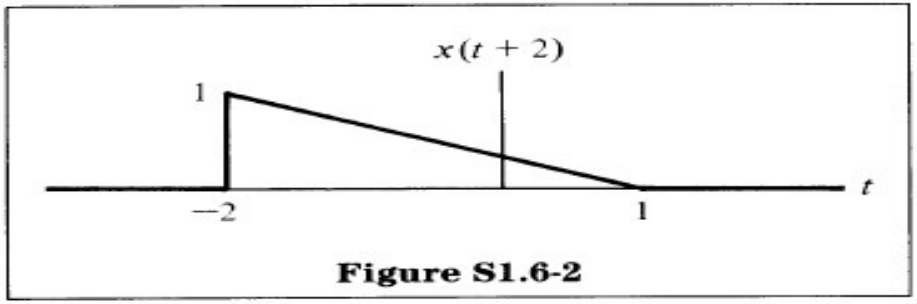
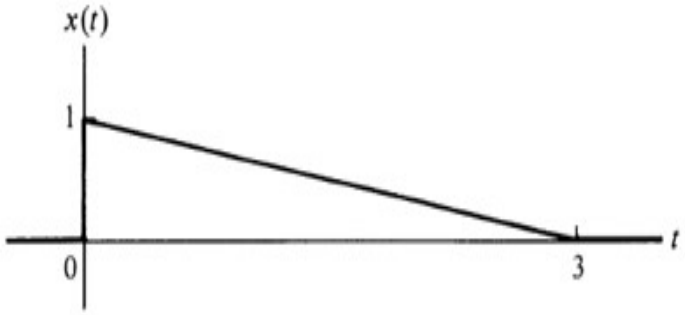


Figure S1.6-2

Acknowledgemet : Lecture slides of Michael D. Adams

Acknowledgement: OCW MIT Resource: Signals and Systems Professor Alan V. Oppenheim

Solution1 Contd...



(c) A scaling by a factor of 2 occurs as well as a time shift.

$0 \le 2t+2 \le 3$

$-2 \le 2t \le 3-2$

$-2 \le 2t \le 1$

$-1 \le t \le \frac{1}{2}$

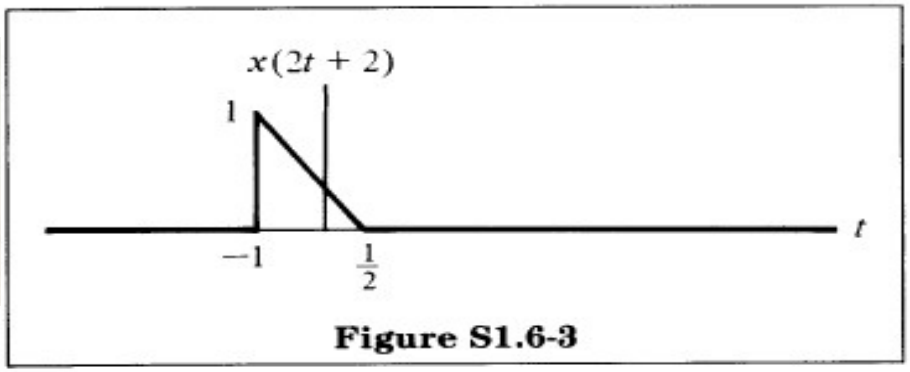
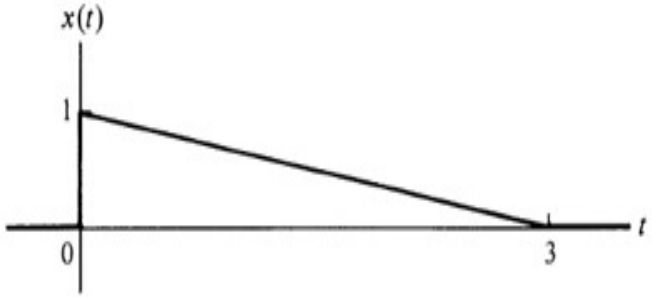


Figure S1.6-3

Note: $a > 1$ induces a compression.

(d) All three effects are combined in this linear scaling.



$0 \le 1-3t \le 3$

$0 \ge 3t-1 \ge -3$

$1 \ge 3t \ge -2$

$\frac{1}{3} \ge t \ge \frac{-2}{3}$

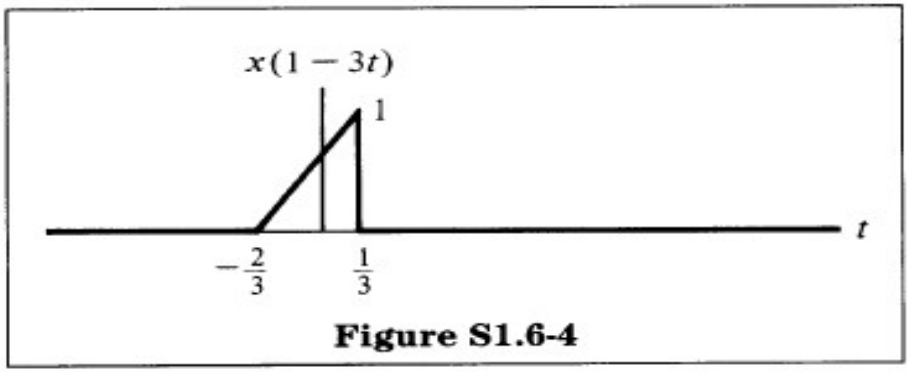


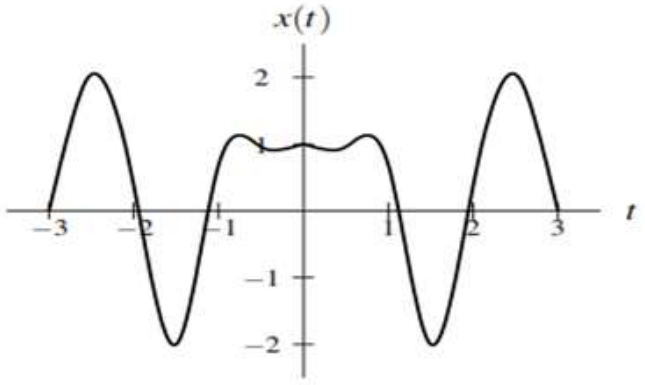
Figure S1.6-4

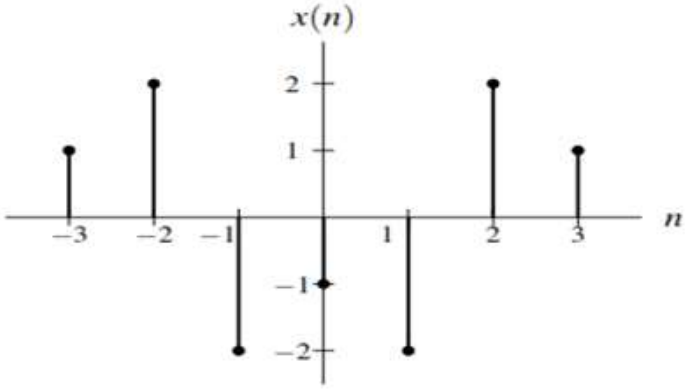
Acknowledgemet : Lecture slides of Michael D. Adams

Acknowledgement: OCW MIT Resource: Signals and Systems Professor Alan V. Oppenheim

Classification of Signals- Even Signals

- A continuous function x is said to be **even** if it satisfies
  - $x(t) = x(-t)$ for all t.
- A discrete function x is said to be **even** if it satisfies
  - $x(n) = x(-n)$ for all n.
- Geometrically, the graph of an even signal is **symmetric** about the origin.
- A few examples of even signals are shown below.





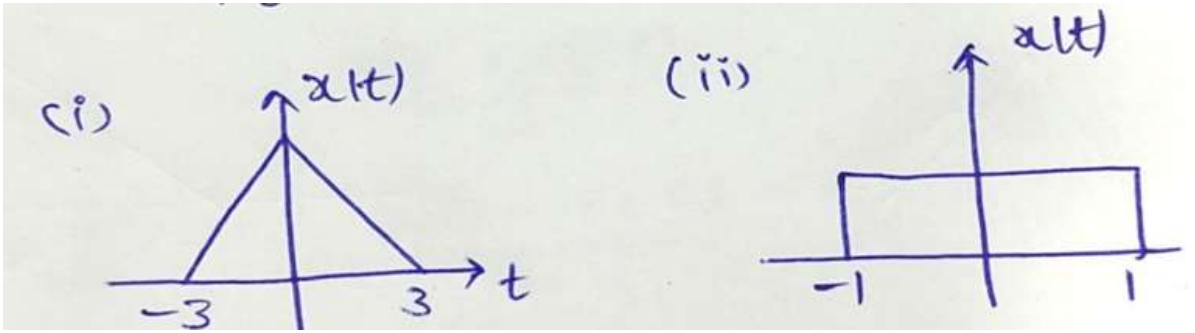
Acknowledgemet : Lecture slides of Michael D. Adams

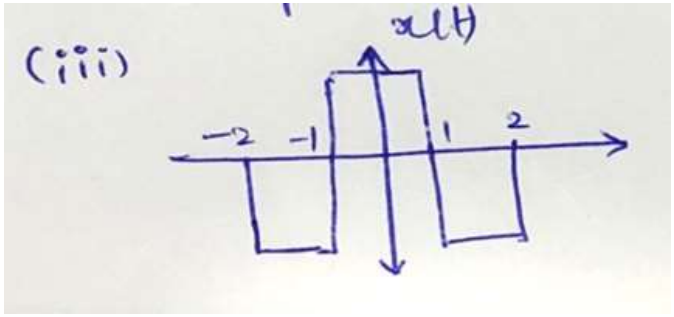
Classification of Signals- Even Signals

- **Even signals:** These are symmetrical (or) mirror image about y-axis.

  i.e. $x(t) = x(-t)$

Eg:-





$(iv)\ x(t) = cos\omega_0 t$

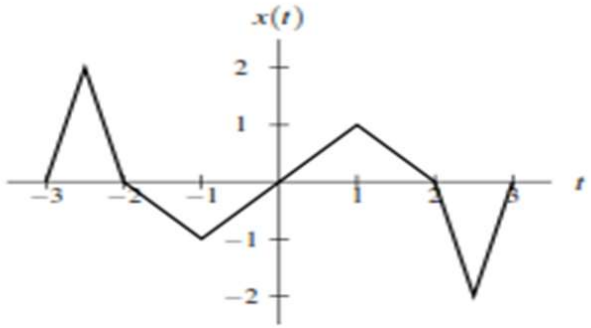
$t \rightarrow -t$

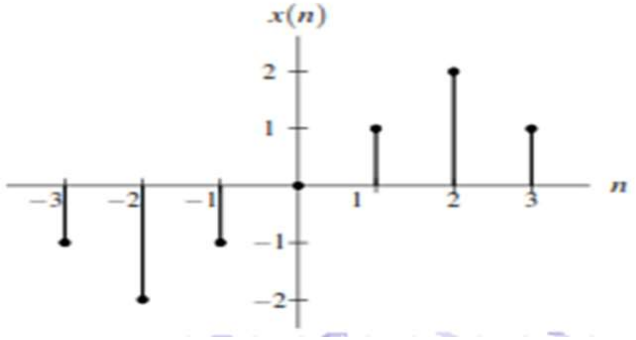
$x(-t) = cos\omega_0(-t) = cos\omega_0 t = x(t)$

Acknowledgemet : Lecture slides of Michael D. Adams

Classification of Signals- Odd Signals

- A function x is said to be **odd** if it satisfies
  - $x(t) = -x(-t)$ for all t.
- A function x is said to be **odd** if it satisfies
  - $x(n) = -x(-n)$ for all n.
- Geometrically, the graph of an odd signal is **antisymmetric** about the origin.
- An odd signal x must be such that $\boldsymbol{x(0) = 0}$.
- A few examples of odd signals are shown below.





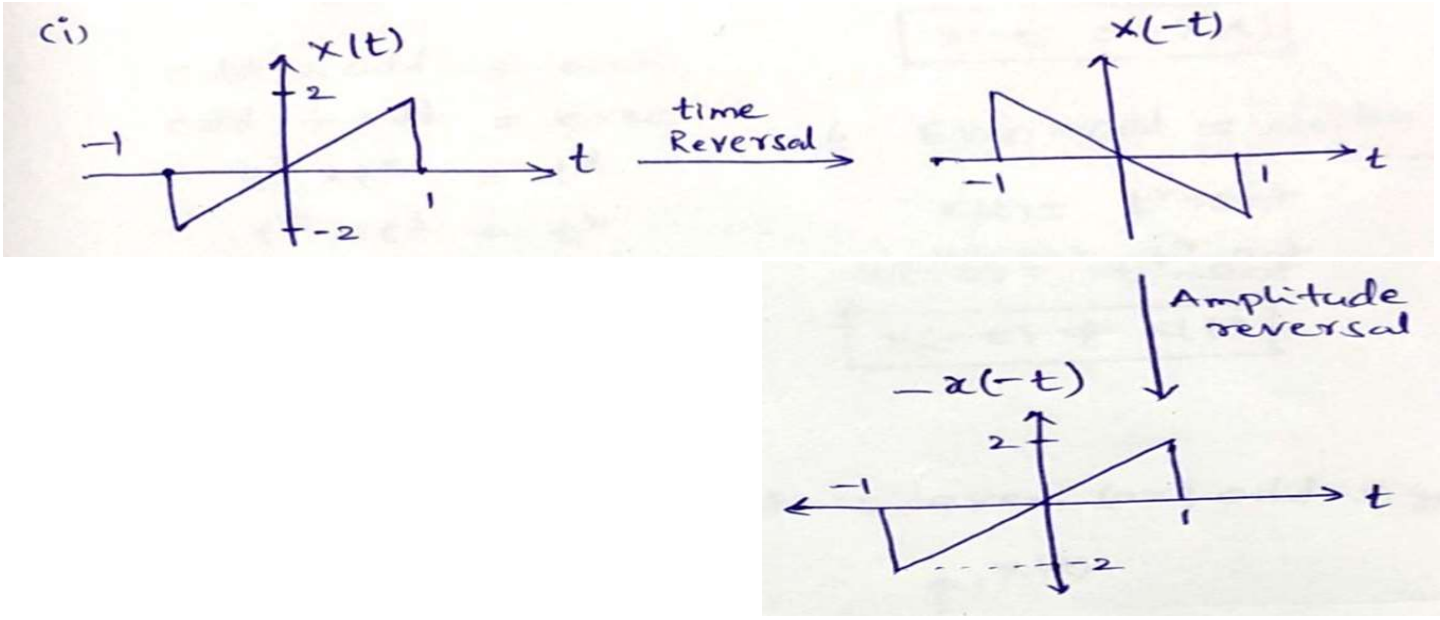
Acknowledgemet : Lecture slides of Michael D. Adams

Classification of Signals- Odd Signals

- **Odd signals:** These are symmetrical (or) mirror image about y-axis.

  i.e. $x(-t) = -x(t)$ or $x(t) = -x(-t)$

Eg:-



Acknowledgemet : Lecture slides of Michael D. Adams

Classification of Signals- Odd Signals

$(ii)\ \ x(t) = Sin\omega_0 t$

$t \rightarrow -t$

$x(-t) = Sin\omega_0(-t) = \ -Sin\omega_0 t = -x(t)$

- Hence it is odd signal

- **Note:** The average value of an odd signal is 0, but converse of this statement is not true.

Acknowledgemet : Lecture slides of Michael D. Adams

Even and Odd Parts of a Signal

- Any signal can be written as a combination of an even and an odd signals
  - Even and Odd components

$$f(t) = \frac{1}{2}(f(t) + f(-t)) + \frac{1}{2}(f(t) - f(-t))$$

$$f_e(t) = \frac{1}{2}(f(t) + f(-t)) \quad \text{even component}$$

$$f_o(t) = \frac{1}{2}(f(t) - f(-t)) \quad \text{odd component}$$

$$f(t) = f_e(t) + f_o(t)$$

Acknowledgemet : Lecture slides of Michael D. Adams

Even and Odd Parts of a Signal

- Any signal can be divided into two parts

$$X(t) = X_{even}(t) + X_{odd}(t)$$

$$X_{even}(t) = \frac{X(t) + X(-t)}{2}$$

$$X_{odd}(t) = \frac{X(t) - X(-t)}{2}$$

Acknowledgemet : Lecture slides of Michael D. Adams

Important properties of Even and Odd functions

- Even function × odd function = odd function
- Odd function × odd function = even function
- Even function × even function = even function
- Area

$$\int_{-a}^{a} f_e(t)dt = 2\int_{0}^{a} f_e(t)dt$$

$$\int_{-a}^{a} f_e(t)dt = 0$$

Acknowledgemet : Lecture slides of Michael D. Adams

Important Properties of Even and Odd Functions

Important points:

1. Even × Even = Even

   $t^2 \times t^4 = t^6$

2. Even × odd = odd

   $t^2 \times t^3 = t^5$

3. odd × odd = even

   odd ÷ odd = even

   $t^3 \times t^5 = t^8$

   $t^5 \div t^3 = t^2$

4. Even + Even = Even

   Even − Even = Even

   $t^2 \pm \cos t \rightarrow$ even

5. odd + odd = odd

   $x(t) = \sin t + t^3$

   $x(t) = x(-t)$

   $x(-t) = -\sin t - t^3$

   $\boxed{x(-t) = -x(t)}$

6. Even + odd = Neither even nor odd

   $x(t) = t^2 + \sin t$

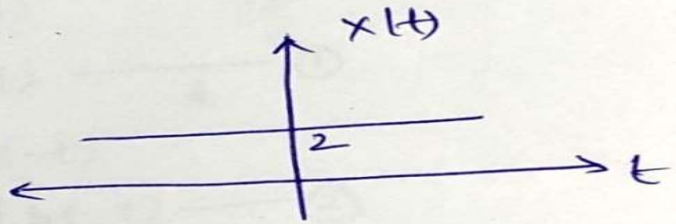
   $x(-t) = t^2 - \sin t$

   $\boxed{x(-t) \neq x(t)}$

Example Problems- Even and Odd Signals

Check whether the given signal is even (or) odd signal.

1. $x(t) = 2$ (DC signal)



$t \to -t$

$x(-t) = 2$

DC signal is an Even signal.

2. $x(t) = \sin(t^2)$

$t \to -t$

$x(-t) = \sin((-t)^2) = \sin t^2 \Rightarrow$ Even signal

3. $x(t) = \sin \pi/2$

$= 1$

$x(-t) = \sin \pi/2 = 1$

DC signal is an Even signal

Example Problems- Even and Odd Signals

4. find $x_{even}(t)$ and $x_{odd}(t)$ of the signal

$$x(t) = 3 - \frac{t^2}{\sin t} + \frac{\cos t}{t} - \frac{\sin^2 t}{t^4} + t^3 \sin^3 t$$

$$E \quad -\frac{E}{O} \quad + \frac{E}{O} \quad - \frac{E}{E} \quad + O \times O$$

$$\downarrow \qquad \downarrow \qquad \downarrow \qquad \downarrow \qquad \downarrow$$

$$E \qquad O \qquad O \qquad E \qquad E$$

$$x_{even}(t) = 3 - \frac{\sin^2 t}{t^4} + t^3 \sin^3 t$$

$$x_{odd}(t) = \frac{-t^2}{\sin t} + \frac{\cos t}{t}$$

Acknowledgemet : Lecture slides of Michael D. Adams

Question2

For each of the following signals, determine whether it is even, odd, or neither.

(a)

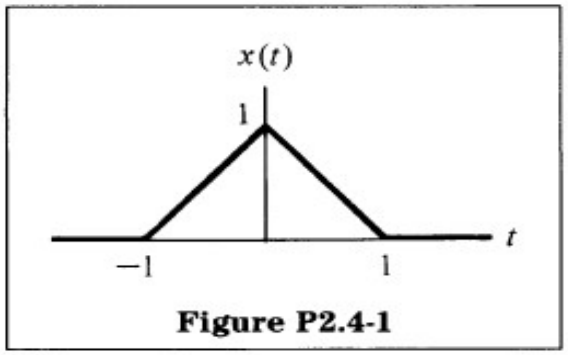


Figure P2.4-1

(b)

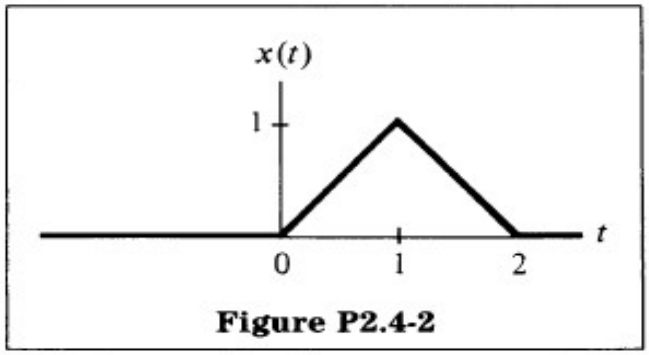


Figure P2.4-2

(c)

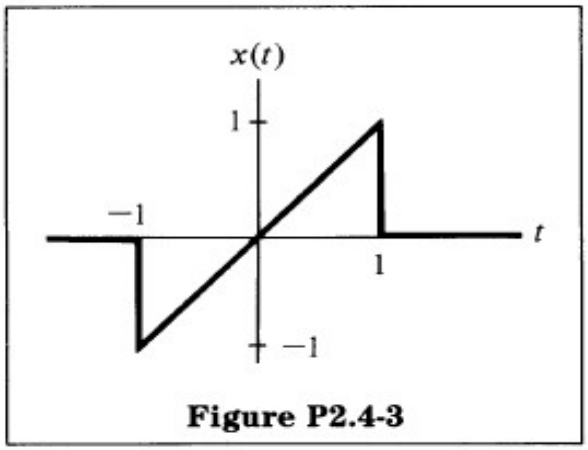


Figure P2.4-3

(d)

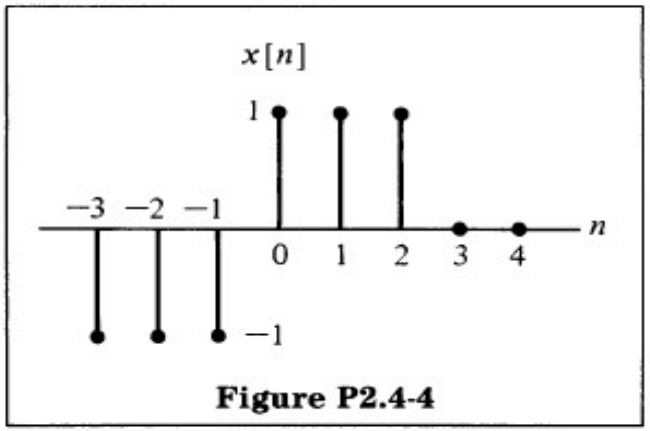


Figure P2.4-4

Acknowledgement: OCW MIT Resource: Signals and Systems Professor Alan V. Oppenheim

Solution2

By definition a signal is even if and only if $x(t) = x(-t)$ or $x[n] = x[-n]$, while a signal is odd if and only if $x(t) = -x(-t)$ or $x[n] = -x[-n]$.

(a) Since $x(t)$ is symmetric about $t = 0$, $x(t)$ is even.

(b) It is readily seen that $x(t) \neq x(-t)$ for all t, and $x(t) \neq -x(-t)$ for all t; thus $x(t)$ is neither even nor odd.

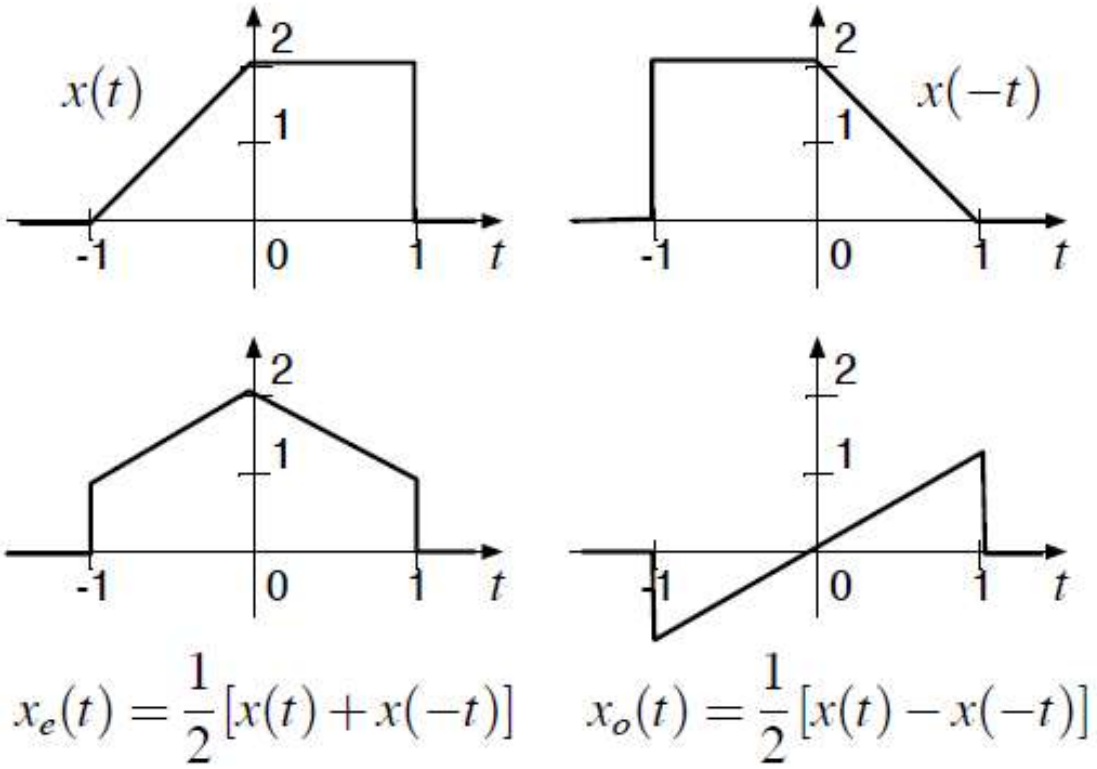
(c) Since $x(t) = -x(-t)$, $x(t)$ is odd in this case.

(d) Here $x[n]$ seems like an odd signal at first glance. However, note that $x[n] = -x[-n]$ evaluated at $n = 0$ implies that $x[0] = -x[0]$ or $x[0] = 0$. The analogous result applies to continuous-time signals. The signal is therefore neither even nor odd.

Acknowledgement: OCW MIT Resource: Signals and Systems Professor Alan V. Oppenheim

Example Problems- Even and Odd Signals

- Example



$$x_e(t) = \frac{1}{2}[x(t) + x(-t)] \qquad x_o(t) = \frac{1}{2}[x(t) - x(-t)]$$

Acknowledgement : Lecture slides of Michael D. Adams

Periodic Signals

- A function x is said to be **periodic** with **period** T (or **T-periodic**) if, for some strictly-positive real constant T, the following condition holds:

$$x(t) = x(t+T) \quad \text{for all } t.$$

- A T-periodic function x is said to have **frequency** $\frac{1}{T}$ and **angular frequency** $\frac{2\pi}{T}$.
- A sequence x is said to be **periodic** with **period** N (or **N-periodic**) if, for some strictly-positive integer constant N, the following condition holds:

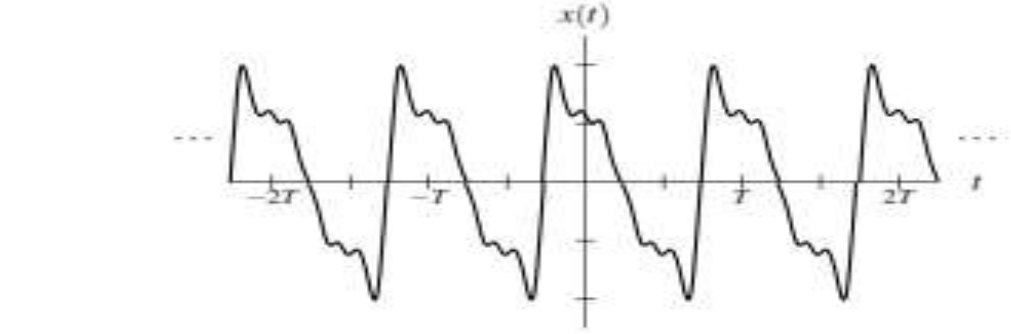
$$x(n) = x(n+N) \quad \text{for all } n.$$

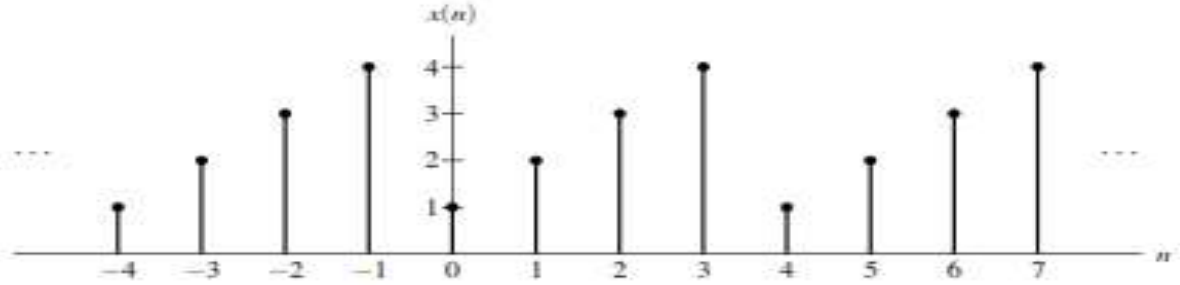
- An N-periodic sequence x is said to have **frequency** $\frac{1}{N}$ and **angular frequency** $\frac{2\pi}{N}$.
- A function/sequence that is not periodic is said to be **aperiodic**.

Acknowledgement : Lecture slides of Michael D. Adams

Example of Periodic Signals

- Some examples of periodic signals are shown below.





Acknowledgement : Lecture slides of Michael D. Adams

Periodic and Non-Periodic Signals

Periodic Signal : A signal is said to be periodic if it repeats a definite pattern of variation after a constant length of time called the time period T_0

$$i.e\ x(t) = x\left[t \pm nT_0\right]$$

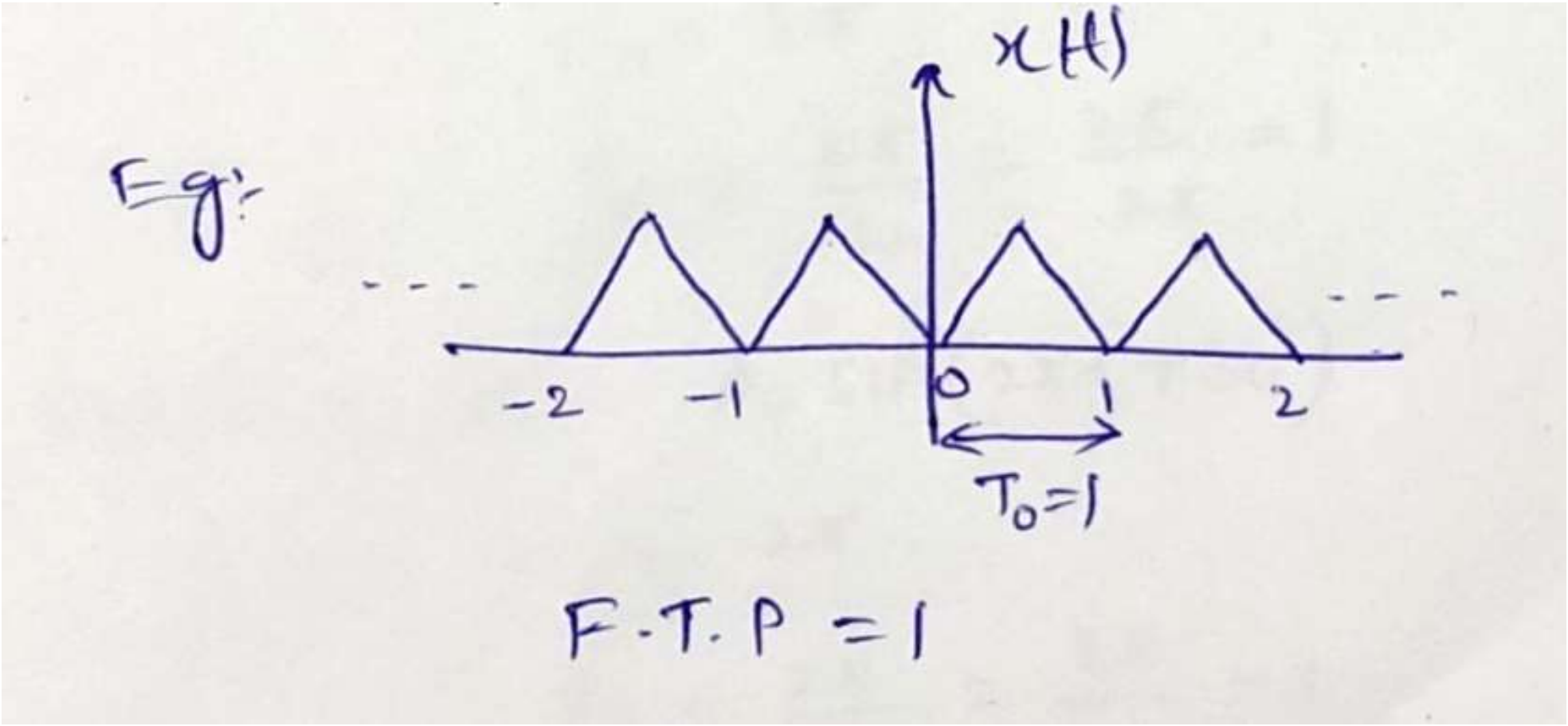
where n is an integer

$T_0 = Fundamental\ Time\ Period$

$$\left(\begin{array}{c} T_0 \neq 0 \\ T_0 \neq \infty \end{array}\right)$$

Fundamental Time Period (FTP): It is the smallest positive and fixed value of the time for which signal is periodic

Find the Fundamental Time period of the Signal



Calculation of Fundamental Time period of the Signals

Q→ Find FTP of signal $x(t)$

$$x(t) = A_0 e^{j\omega_0 t}$$

Sol Let T_0 be the FTP of the signal

i.e $x(t) = x(t+T_0)$

$$x(t+T_0) = A_0 e^{j\omega_0(t+T_0)}$$

$$A_0 e^{j\omega_0 t} = A_0 e^{j\omega_0(t+T_0)}$$

$$A_0 e^{j\omega_0 t} = A_0 e^{j\omega_0 t} e^{j\omega_0 T_0}$$

$$e^{j\omega_0 T_0} = 1 = e^{j2\pi k}$$

$$j\omega_0 T_0 = j2\pi k$$

$$T_0 = \frac{2\pi k}{\omega_0}$$ (make $k=1$ to make T_0 smallest)

$$\boxed{T_0 = \frac{2\pi}{\omega_0}}$$

Calculation of Fundamental Time period of the Signals

Q → Find FTP of following signal

(1) $x_1(t) = A_0 \sin(2\pi t)$

$\omega_0 = 2\pi$

$T_0 = \frac{2\pi}{\omega_0} = \frac{2\pi}{2\pi} = 1$

(2) $x_2(t) = A_0 \sin(2\pi t + 30^\circ)$

$\omega_0 = 2\pi$

$T_0 = \frac{2\pi}{\omega_0} = \frac{2\pi}{2\pi} = 1$

(3) $x_3(t) = -x_1(t)$

$= -A_0 \sin(2\pi t)$

$\omega_0 = 2\pi$

$T_0 = \frac{2\pi}{\omega_0} = \frac{2\pi}{2\pi} = 1$

Calculation of Fundamental Time period of the Signals

(4) $x_4(-t) = x_1(-t)$

$= A_0 \sin 2\pi(-t)$

$= -A_0 \sin 2\pi t$

$\omega_0 = 2\pi$

$T_0 = \frac{2\pi}{\omega_0} = \frac{2\pi}{2\pi} = 1$

(5) $x_5(t) = A_0 + x_1(t)$

$= A_0 + A_0 \sin 2\pi t$

$\omega_0 = 2\pi$

$T_0 = \frac{2\pi}{\omega_0} = \frac{2\pi}{2\pi} = 1$

Calculation of Fundamental Time period of the Signals

(6) $x_6(t) = x_1(t - t_0)$

$= A_0 \sin(2\pi(t - t_0))$

$\omega_0 = 2\pi$

$T_0 = \frac{2\pi}{\omega_0} = \frac{2\pi}{2\pi} = 1$

Note: Time period of the signal is unaffected by Time shifting, Time reversal, Amplitude reversal, Amplitude shifting and Change in phase of signal.

Calculation of Fundamental Time period of the Signals

(7) $x_7(t) = \sin^2(4\pi t)$

$$= \frac{1 - \cos 8\pi t}{2}$$

$$\omega_0 = 8\pi$$

$$T_0 = \frac{2\pi}{\omega_0} = \frac{2\pi}{8\pi} = \frac{1}{4}$$

Sum of Periodic Functions

- **Sum of periodic functions.** Let x_1 and x_2 be periodic functions with fundamental periods T_1 and T_2, respectively. Then, the sum $y = x_1 + x_2$ is a periodic function if and only if the ratio T_1/T_2 ***is a rational number*** (i.e., the quotient of two integers). Suppose that $T_1/T_2 = q/r$ where q and r are integers and ***coprime*** (i.e., have no common factors), then the fundamental period of y is rT_1 (or equivalently, qT_2, since $rT_1 = qT_2$). (Note that rT_1 is simply the least common multiple of T_1 and T_2.)
- Although the above theorem only directly addresses the case of the sum of two functions, the case of N functions (where $N > 2$) can be handled by applying the theorem repeatedly $N - 1$ times.

Acknowledgement : Lecture slides of Michael D. Adams

Calculation of FTP of sum of two or more signals

Note: The sum of 2 or more than 2 periodic signals will be periodic if ratios of these Fundamental time period (or) frequencies are rational.

i.e $x(t) = x_1(t) + x_2(t)$

$T_1, f_1, \omega_1 \qquad T_2, f_2, \omega_2$

↳ $\frac{T_1}{T_2}$ or $\frac{\omega_1}{\omega_2}$ or $\frac{f_1}{f_2}$ must be rational

→ $T_0 = LCM(T_1, T_2)$

→ $F_0 = HCF(f_1, f_2)$

Example problems on Fundamental Time Period

Q Find FTP of signal if it is periodic :-

(i) $x(t) = \sin 2t + \cos 3\pi t$

Sol: $\omega_1 = 2$, $\omega_2 = 3\pi$

$$\frac{\omega_1}{\omega_2} = \frac{2}{3\pi} \quad \text{(Irrational)}$$

Hence it is non-periodic

(ii) $x(t) = \sin 2\pi t + \cos \sqrt{2}\pi t$

Sol: $\omega_1 = 2\pi$

$\omega_2 = \sqrt{2}\pi$

$$\frac{\omega_1}{\omega_2} = \frac{2\pi}{\sqrt{2}\pi} = \sqrt{2} \quad \text{(Irrational)}$$

Hence it is non-periodic

Example problems on FTP

(iii) $x(t) = \sin 4\pi t + \sin 7\pi t$

Sol:

$\omega_1 = 4\pi$

$\omega_2 = 7\pi$

$\frac{\omega_1}{\omega_2} = \frac{4\pi}{7\pi} = \frac{4}{7}$ (Rational no.)

Hence it is periodic. Then calculate T_0.

Example problems on FTP

Method 1:

$$\omega_0 = HCF(\omega_1, \omega_2)$$

$$= HCF(4\pi, 7\pi)$$

$$\omega_0 = \pi$$

$$T_0 = \frac{2\pi}{\omega_0} = 2$$

Method 2:

$$T_1 = \frac{2\pi}{\omega_1} = \frac{2\pi}{4\pi} = \frac{1}{2}$$

$$T_1 = \frac{2\pi}{\omega_2} = \frac{2\pi}{7\pi} = \frac{2}{7}$$

$$T_0 = LCM(T_1, T_2)$$

$$= LCM\left(\frac{1}{2}, \frac{2}{7}\right)$$

$$= \frac{LCM[1,2]}{HCF[2,7]} = \frac{2}{1} = 2$$

$$LCM\left[\frac{p_1}{q_1}, \frac{p_2}{q_2}\right] = \frac{LCM[p_1, p_2]}{HCF[q_1, q_2]}$$

Acknowledgement : Lecture slides of Michael D. Adams

Signal Energy and Power

- The **energy** E contained in the signal x is given by

$$E = \int_{-\infty}^{\infty} |x(t)|^2 \, dt.$$

- A signal with finite energy is said to be an **energy signal**.
- The **average power** P contained in the signal x is given by

$$P = \lim_{T \to \infty} \frac{1}{T} \int_{-T/2}^{T/2} |x(t)|^2 \, dt.$$

- A signal with (nonzero) finite average power is said to be a **power signal**.

Acknowledgement : Lecture slides of Michael D. Adams

Energy and Power of signals

Energy of $x(t) = \int_{-\infty}^{\infty} |x(t)|^2 \, dt$

$$E = \int_{-\infty}^{\infty} |x(t)|^2 \, dt$$

power of $x(t)$

$$P = \begin{cases} \frac{1}{T_0} \int_{-T_0/2}^{T_0/2} |x(t)|^2 dt & \text{: for periodic sig} \\ \lim_{T_0 \to \infty} \int_{-T_0/2}^{T_0/2} |x(t)|^2 dt & \text{: for non periodic sig.} \end{cases}$$

For an energy signal, { energy should be finite
power should be zero

Energy signals are absolutely integrable

i.e $\boxed{\int_{-\infty}^{\infty} |x(t)| \, dt < \infty}$

Find the Energy and Power of the given signal $x(t) = e^{-2t}\,u(t)$

a) Given that, $x(t) = e^{-2t}\,u(t)$

Here, $x(t) = e^{-2t}\,u(t)$; for all t

$\therefore\ x(t) = e^{-2t}$; for $t \geq 0$

$$\therefore \int_{-T}^{T} |x(t)|^2\,dt = \int_{0}^{T} \left(\left|e^{-2t}\right|\right)^2 dt = \int_{0}^{T} \left(e^{-2t}\right)^2 dt = \int_{0}^{T} e^{-4t}\,dt = \left[\frac{e^{-4t}}{-4}\right]_0^T$$

$$= \left[\frac{e^{-4T}}{-4} - \frac{e^{0}}{-4}\right] = \left[\frac{1}{4} - \frac{e^{-4T}}{4}\right]$$

$$\text{Energy, } E = \lim_{T \to \infty} \int_{-T}^{T} |x(t)|^2\,dt = \lim_{T \to \infty} \left[\frac{1}{4} - \frac{e^{-4T}}{4}\right]$$

$$= \frac{1}{4} - \frac{e^{-\infty}}{4} = \frac{1}{4} - \frac{0}{4} = \frac{1}{4} \text{ joules}$$

$$\text{Power, } P = \lim_{T \to \infty} \frac{1}{2T} \int_{-T}^{T} |x(t)|^2\,dt = \lim_{T \to \infty} \frac{1}{2T}\left[\frac{1}{4} - \frac{e^{-4T}}{4}\right]$$

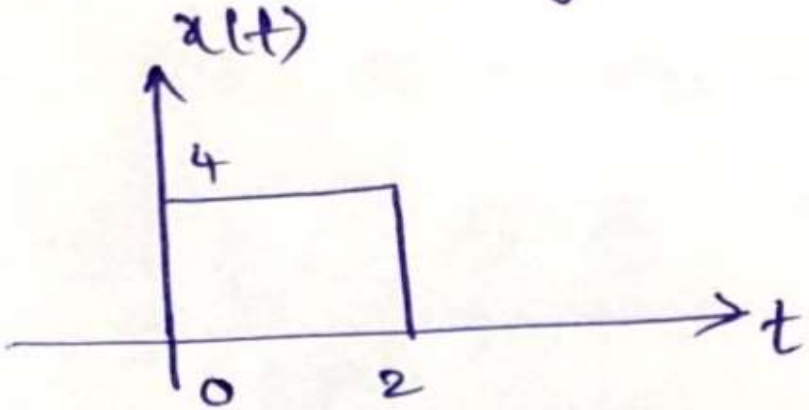
$$= \frac{1}{\infty}\left[\frac{1}{4} - \frac{e^{-\infty}}{4}\right] = 0 \times \left[\frac{1}{4} - 0\right] = 0$$

Since energy is constant and power is zero, the given signal is an energy signal.

Find the Energy of a given signal

Q → Calculate energy of signal.

(i)



Sol →

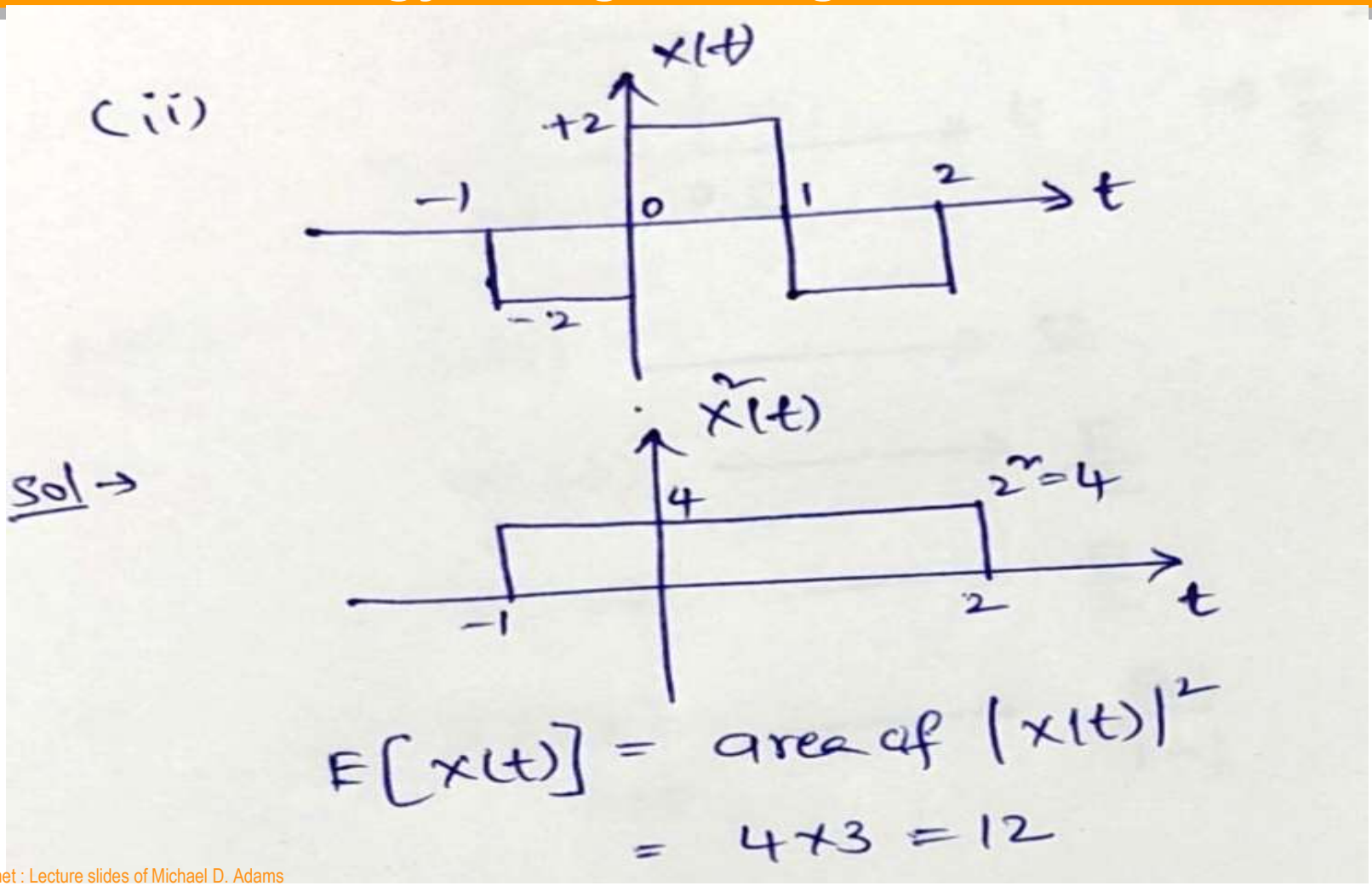
$$E = \int_{-\infty}^{\infty} |x(t)|^2 dt = \int_{0}^{2} 4^2 dt = 32$$

(OR)

$$E = \text{Area of } |x(t)|^2 = 16 \times 2 = 32$$

Acknowledgemet : Lecture slides of Michael D. Adams

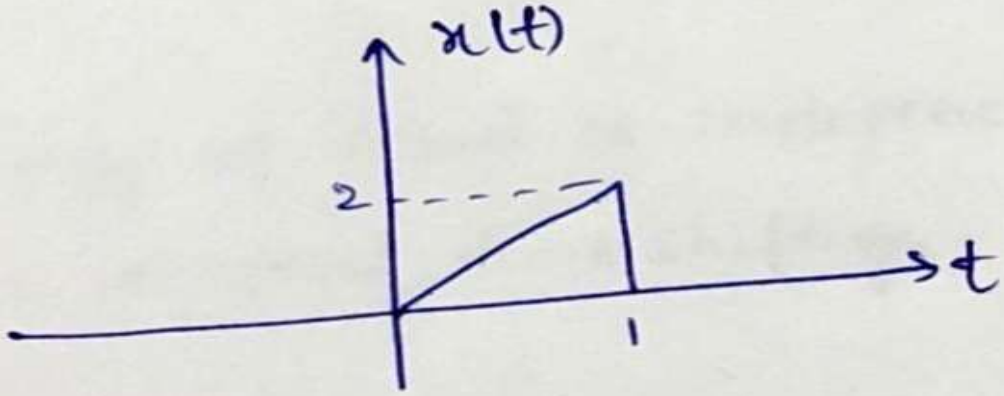
Find the Energy of a given signal



Acknowledgemet : Lecture slides of Michael D. Adams

Find the energy of a given signal

(iii)



Sol:- Here $x(t) = 2t$

$y = mx$ form

$m = \text{slope} = \frac{2}{1} = 2$

$$E = \int_{-\infty}^{\infty} |x(t)|^2 dt$$

$$= \int_{0}^{1} (2t)^2 dt = \frac{4}{3}$$

Acknowledgemet : Lecture slides of Michael D. Adams

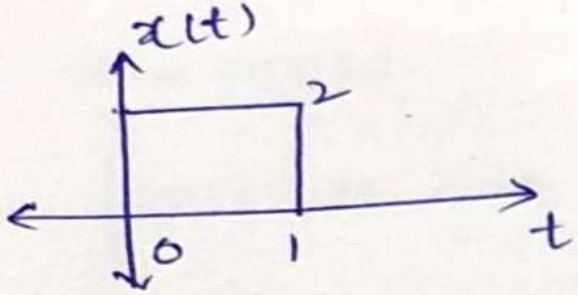
Find the Energy of a given signal

Q $x(t)$ signal has Energy E.

Find the energy of the signal $x(2t)$

a) $\frac{E}{4}$ b) $\frac{E}{2}$ c) $2E$ d) E

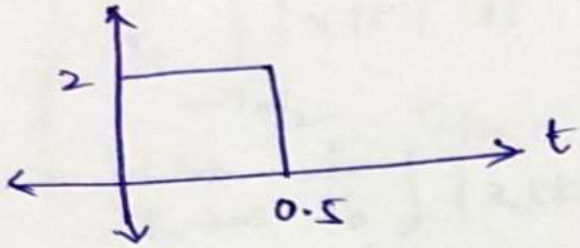
Sol



$E \rightarrow 4$

E = Area of $|x(t)|^2$

$= 4 \times 1 = 4$



$E' \rightarrow 2$

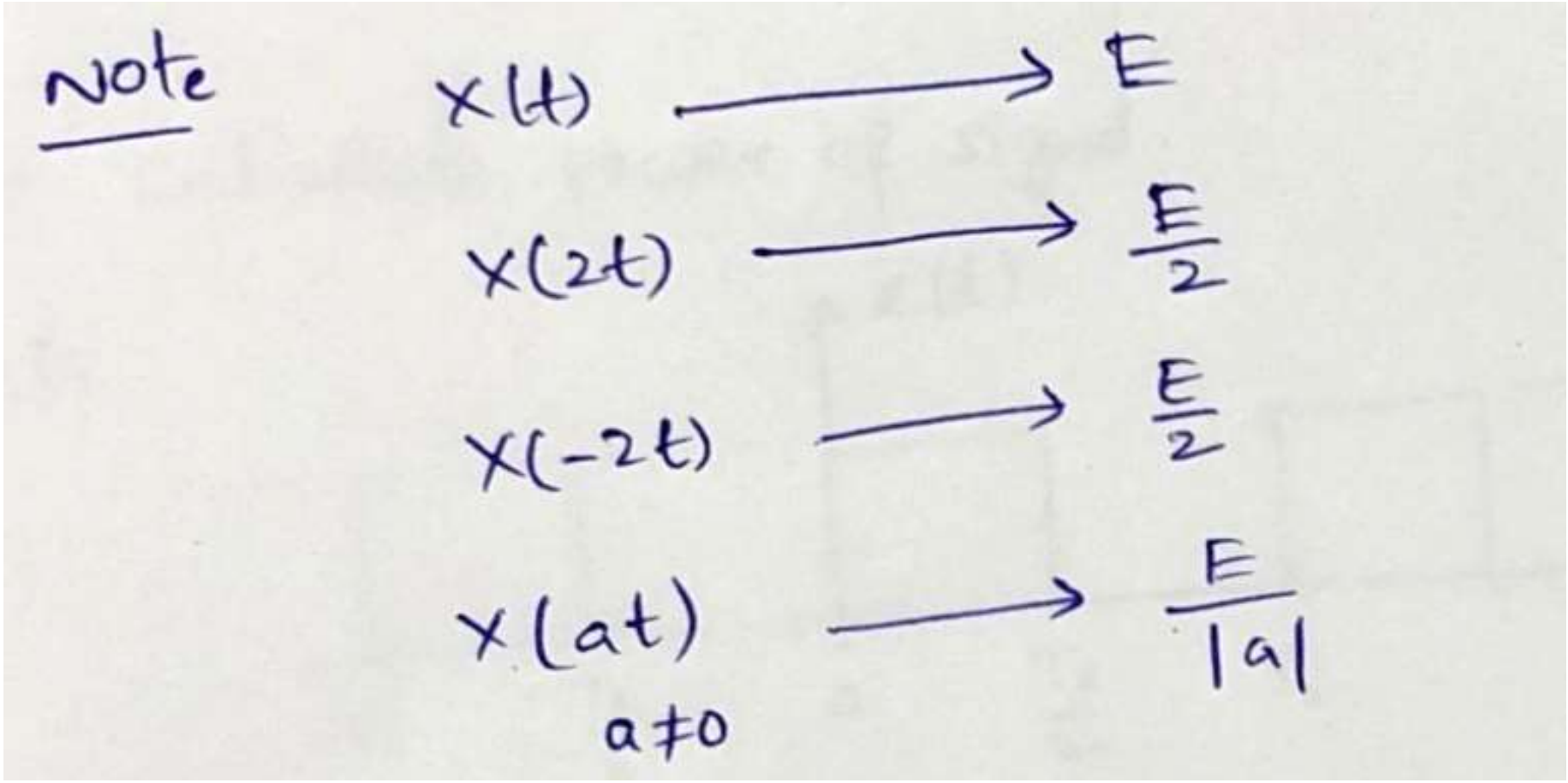
$\Rightarrow \frac{E}{2}$

E = Area of $|x(2t)|^2$

$= 4 \times 0.5 = 2$

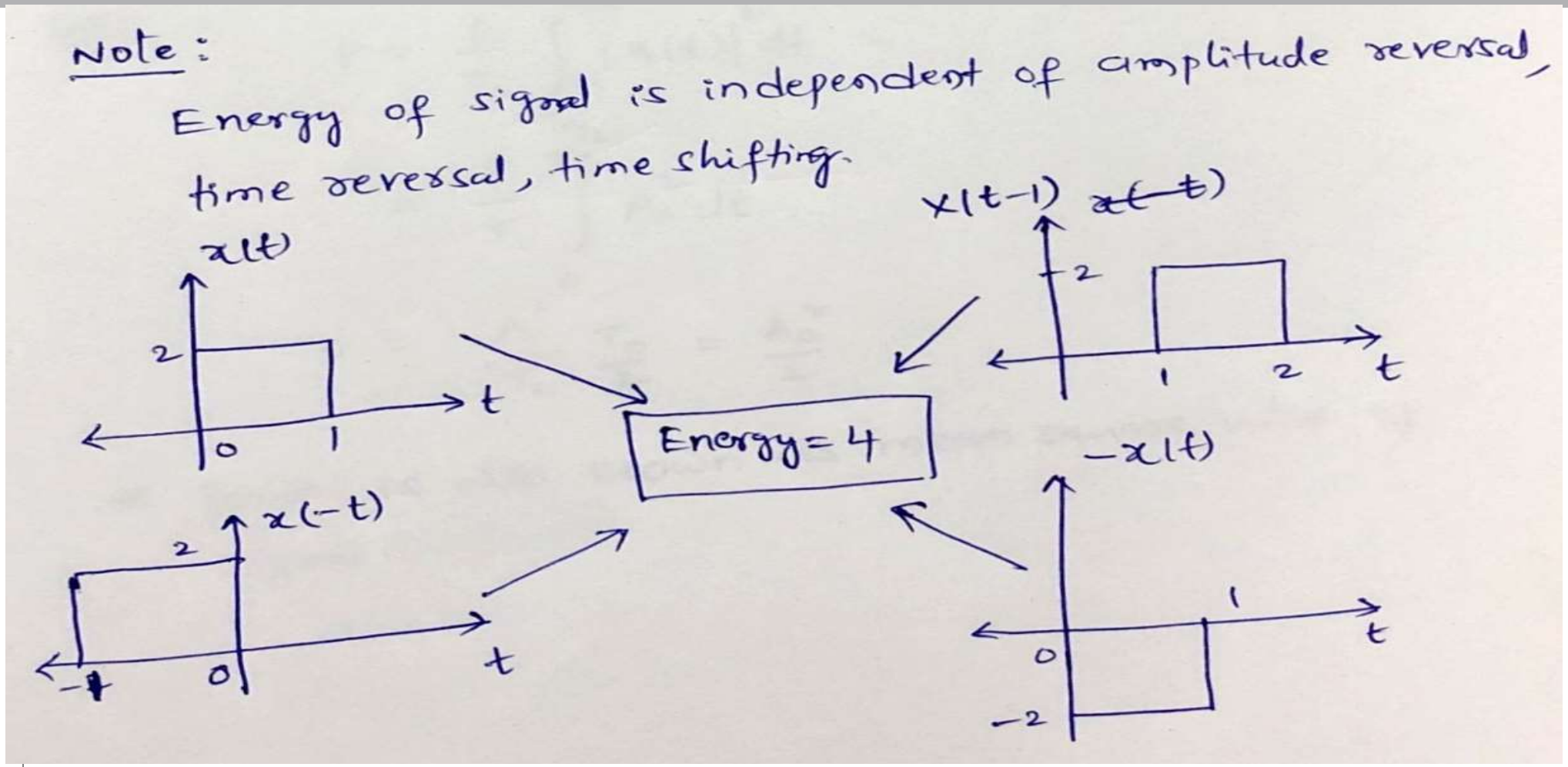
Important points to remember Related to Energy of a give Signal

If a signal x(t) has Energy E, then for the signal x(2t) the Energy reduces by a factor of 2



Acknowledgemet : Lecture slides of Michael D. Adams

Important Note to remember Related to Energy of a give Signal



Acknowledgemet : Lecture slides of Michael D. Adams

Finding the Power of a given Signal

Power signal

1) For this signal power should be finite and Energy should be ∞.

2) periodic power signals are absolutely integrable over their time period.

i.e $\int_{T_0} |x(t)|\,dt < \infty$ periodic power s/g

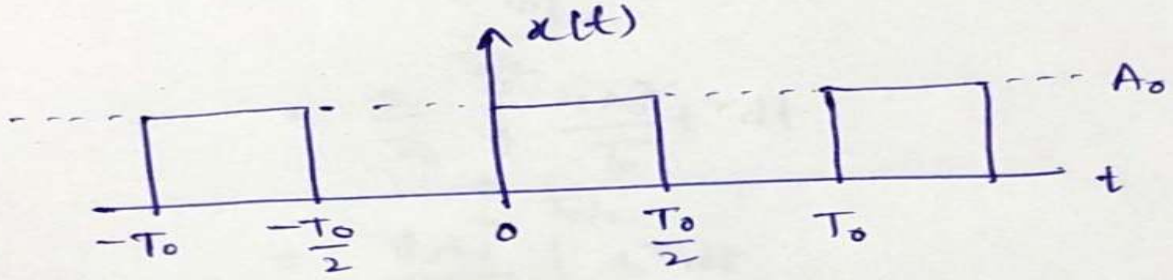
$$P = \begin{cases} \frac{1}{T_0} \int_{-T_0/2}^{T_0/2} |x(t)|^2 dt, & \text{For periodic signal} \\ \lim_{T_0 \to \infty} \frac{1}{T_0} \int_{-T_0/2}^{T_0/2} |x(t)|^2 dt, & \text{for non-periodic signal.} \end{cases}$$

Acknowledgemet : Lecture slides of Michael D. Adams

Find the Power of a given Pulse Signal

Q → Calculate power of signal.

(i)



Sol:

$$P = \frac{1}{T_0}\int_{-T_0/2}^{T_0/2} |x(t)|^2 dt$$

$$= \frac{1}{T_0}\int_{-T_0/2}^{0} A_0^2 dt$$

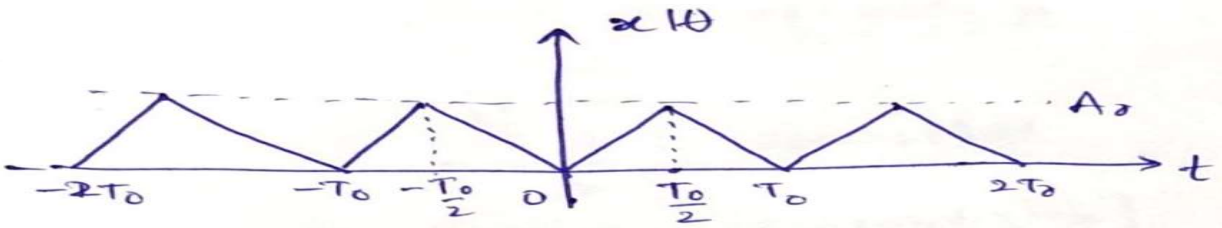
$$P = \frac{A_0^2}{T_0}\frac{T_0}{2} = \frac{A_0^2}{2}$$

* Power is also known as mean square value of signal.

Acknowledgemet : Lecture slides of Michael D. Adams

Find the power of a given triangular waveform

(ii)



Sol

$$P = \frac{1}{T_0}\int_{-T_0/2}^{T_0/2} |x(t)|^2 dt, \quad x(t) = mt$$

$$m = \frac{A_0 - 0}{\frac{T_0}{2} - 0} = \frac{2A_0}{T_0}$$

$$= \frac{1}{T_0}\int_{-T_0/2}^{T_0/2} \left|\left(\frac{2A_0}{T_0}\right)t\right|^2 dt$$

$$= \frac{2}{T_0}\int_{0}^{T_0/2} \frac{4A_0^2}{T_0^2} t^2 dt$$

$$= \frac{8A_0^2}{T_0^3}\int_{0}^{T_0/2} t^2 dt$$

$$= \frac{8A_0^2}{T_0^3} \cdot \frac{T_0^3}{8 \times 3}$$

$$\boxed{P = \frac{A_0^2}{3}}$$

Acknowledgemet : Lecture slides of Michael D. Adams

Find the power of a sinusoidal signal

(iii) $x(t) = A_0 \sin \omega_0 t$

Sol

$$P = \frac{1}{T_0} \int_{-T_0/2}^{T_0/2} (A_0 \sin \omega_0 t)^2 dt$$

$$P = \frac{A_0^2}{T_0} \int_{-T_0/2}^{T_0/2} \left(\frac{1 - \cos 2\omega_0 t}{2}\right) dt$$

$$= \frac{2A_0^2}{2T_0} \int_{0}^{T_0/2} (1 - \cos 2\omega_0 t)\, dt$$

$$= \frac{A_0^2}{T_0} \left[\frac{T_0}{2} - \left(\frac{\sin 2\omega_0 t}{2\omega_0}\right)_0^{T_0/2} \right]$$

$$= \frac{A_0^2}{T_0} \left[\frac{T_0}{2} - \frac{\sin 2\omega_0 \frac{T_0}{2}}{2\omega_0} \right]$$

$$= \frac{A_0^2}{T_0} \cdot \frac{T_0}{2}$$

$$\boxed{P = \frac{A_0^2}{2}}$$

Note :- We know that RMS of $A_0 \sin \omega_0 t = \frac{A_0}{\sqrt{2}}$

$$P = RMS^2 = \left(\frac{A_0}{\sqrt{2}}\right)^2 = \frac{A_0^2}{2}$$

Acknowledgemet : Lecture slides of Michael D. Adams

Previous GATE Questions and Answers

GATE 2013

Q. 3 For a periodic signal $v(t) = 30\sin 100t + 10\cos 300t + 6\sin(500t + \pi/4)$, the fundamental frequency in rad/s

(A) 100

(B) 300

(C) 500

(D) 1500

Acknowledgemet : Lecture slides of Michael D. Adams

Previous GATE Questions and Answers

Option (A) is correct.

Given, the signal

$$v(t) = 30\sin 100t + 10\cos 300t + 6\sin\left(500t + \tfrac{\pi}{4}\right)$$

So we have

$$\omega_1 = 100 \text{ rad/s},\ \omega_2 = 300 \text{ rad/s and } \omega_3 = 500 \text{ rad/s}$$

Therefore, the respective time periods are

$$T_1 = \frac{2\pi}{\omega_1} = \frac{2\pi}{100}\text{sec},\ T_2 = \frac{2\pi}{\omega_2} = \frac{2\pi}{300}\text{sec and } T_3 = \frac{2\pi}{500}\text{sec}$$

So, the fundamental time period of the signal is

$$\text{L.C.M. } (T_1, T_2 T_3) = \frac{LCM(2\pi, 2\pi, 2\pi)}{HCF(100, 300, 500)}$$

or,

$$T_0 = \frac{2\pi}{100}$$

Hence, the fundamental frequency in rad/sec is $\omega_0 = \frac{2\pi}{10} = 100 \text{ rad/s}$

Acknowledgemet : Lecture slides of Michael D. Adams

GATE 2005

Q. 69 The power in the signal $s(t) = 8\cos(20\pi t - \frac{\pi}{2}) + 4\sin(15\pi t)$ is

(A) 40

(B) 41

(C) 42

(D) 82

Acknowledgemet : Lecture slides of Michael D. Adams

Solution

Sol. 69 Option (A) is correct.

$$s(t) = 8\cos\left(\frac{\pi}{2} - 20\pi t\right) + 4\sin 15\pi t$$

$$= 8\sin 20\pi t + 4\sin 15\pi t$$

Here $A_1 = 8$ and $A_2 = 4$. Thus power is

$$P = \frac{A_1^2}{2} + \frac{A_2^2}{2} = \frac{8^2}{2} + \frac{4^2}{2} = 40$$

Acknowledgemet : Lecture slides of Michael D. Adams