Kinematics Of Mechanisms and Robots

MED 151

By
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2026

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MED 151 Kinematics Of Mechanisms Robots

3 Cr. hrs. =[2 Lect. + 2 Tut + 1Lab]

Prerequisite: BAS 031

Kinematics fundamental of rigid bodies, geometry of motion and mechanisms and Planar robots, kinematics of linkage mechanisms, Direct and inverse kinematics of Planar robots, cam - follower mechanisms: design and analysis, standard cams and Equivalent mechanisms , kinematics, geometry and assembly conditions of gear Trains: simple, compound and planetary gear trains, Belts and chains,computer Simulation and case studies. Course Project.

مكص ١٥١ كينماتيكا الآليات والروبوتات

كينماتيكا الحركة المستوية للاجسام الجاسئة، مدخل للاليات والروبوتات المستوية، التحليل الكينماتى للاليات والروبوتات المستوية: الوضع، السرعة، العجلة، مجموعات التروس: البسيطة المركبة، الكوكبية، تصميم وتحليل الكامات، كينماتيكا السيور والجنازير، تطبيقات استخدام الحاسوب فى تحليل الاليات والروبوتات، مشروع مقرر.

References:

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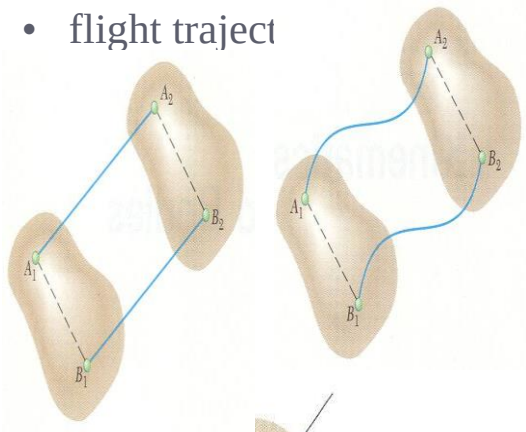
Assessment:

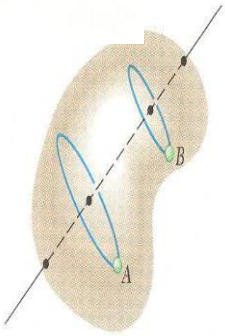
Final Exam: 50%, Quizzes: 20%, Year work: 20%, Experimental /Oral: 10%

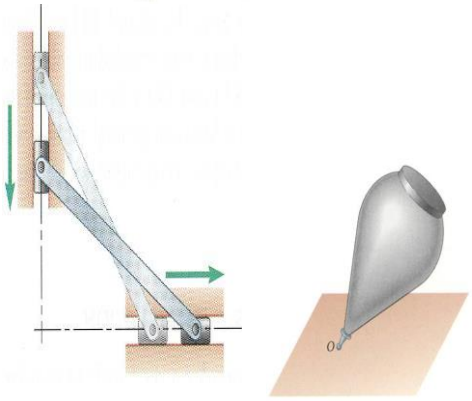
CHAPTER.1.
Kinematics of Rigid Bodies

Introduction

- Kinematics of rigid bodies: Kinematics is the branch of dynamics which describes the motion of bodies without reference to the forces which either cause the motion or are generated as a result of the motion. Kinematics is often described as the "geometry of motion." Some engineering applications of kinematics include the design of cams, gears, linkages, and other machine elements to control or produce certain desired motions, and the calculation of
- flight traject ıft, rockets, and spacecraft.



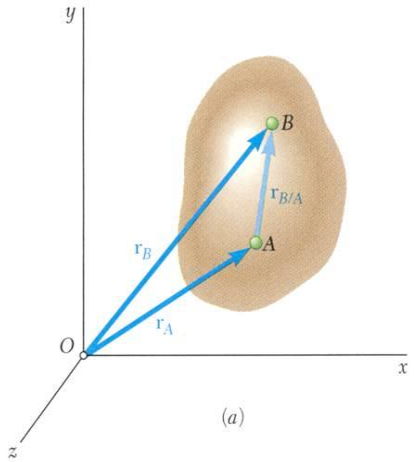




- Classification of rigid body motions:
  - translation:
    - rectilinear translation
    - curvilinear translation
  - rotation about a fixed axis
  - general plane motion
  - general motion
    - motion about a fixed point

Translation

- Consider rigid body in translation:
  - direction of any straight line inside the body is constant,
  - all particles forming the body move in parallel lines.



- For any two particles in the body,

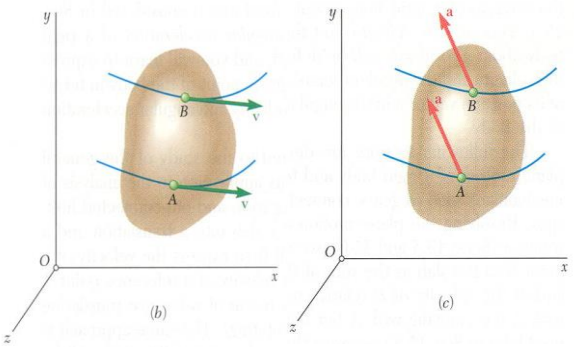
$$\vec{r}_B = \vec{r}_A + \vec{r}_{B/A}$$

- Differentiating with respect to time,

$$\dot{\vec{r}}_B = \dot{\vec{r}}_A + \dot{\vec{r}}_{B/A} = \dot{\vec{r}}_A$$

$$\vec{v}_B = \vec{v}_A$$

All particles have the same velocity.



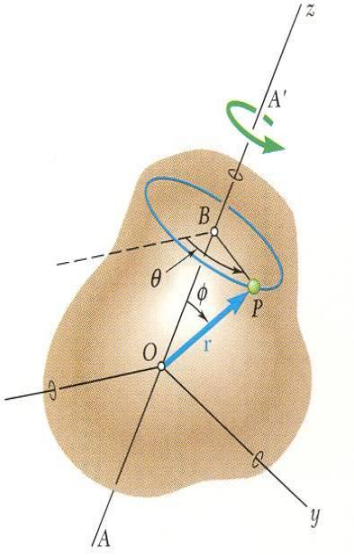
- Differentiating with respect to time again,

$$\ddot{\vec{r}}_B = \ddot{\vec{r}}_A + \ddot{\vec{r}}_{B/A} = \ddot{\vec{r}}_A$$

$$\vec{a}_B = \vec{a}_A$$

All particles have the same acceleration.

Rotation About a Fixed Axis. Velocity



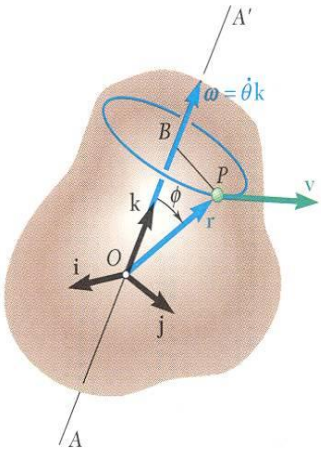
- Consider rotation of rigid body about a fixed axis *AA'*

- Velocity vector of the particle *P* is tangent to the path with magnitude

$$\vec{v} = d\vec{r}/dt \qquad v = ds/dt$$

$$\Delta s = (BP)\Delta\theta = (r\sin\phi)\Delta\theta$$

$$v = \frac{ds}{dt} = \lim_{\Delta t\to 0}(r\sin\phi)\frac{\Delta\theta}{\Delta t} = r\dot{\theta}\sin\phi$$

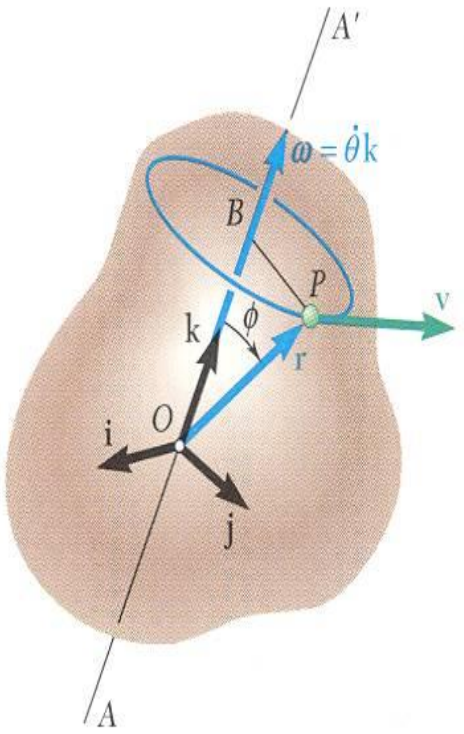


- The same result is obtained from

$$\vec{v} = \frac{d\vec{r}}{dt} = \vec{\omega}\times\vec{r}$$

$$\vec{\omega} = \omega\hat{k} = \dot{\theta}\hat{k} = \textit{angular velocity}$$

Acceleration



- Differentiating to determine the acceleration,

$$\vec{a} = \frac{d\vec{v}}{dt} = \frac{d}{dt}\left(\vec{\omega}\times\vec{r}\right)$$

$$= \frac{d\vec{\omega}}{dt}\times\vec{r} + \vec{\omega}\times\frac{d\vec{r}}{dt}$$

$$= \frac{d\vec{\omega}}{dt}\times\vec{r} + \vec{\omega}\times\vec{v}$$

- $\dfrac{d\vec{\omega}}{dt} = \vec{\alpha} = \text{angular acceleration}$

$$= \alpha\hat{k} = \dot{\omega}\hat{k} = \ddot{\theta}\hat{k}$$

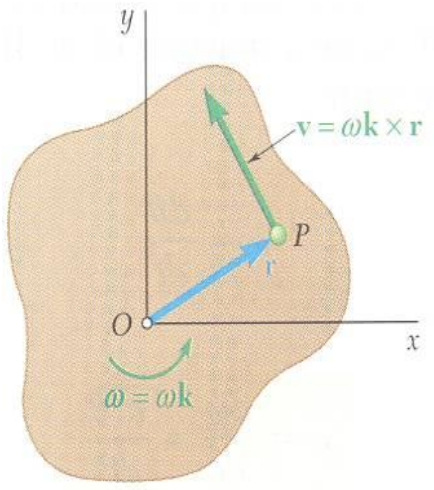
- Acceleration of P is combination of two vectors,

$$\vec{a} = \vec{\alpha}\times\vec{r} + \vec{\omega}\times(\vec{\omega}\times\vec{r}\,)$$

$$\vec{\alpha}\times\vec{r} = \text{tangential acceleration component}$$

$$\vec{\omega}\times(\vec{\omega}\times\vec{r}\,) = \text{radial acceleration component}$$

Representative Slab



- Consider the motion of a representative slab in a plane perpendicular to the axis of rotation.
- Velocity of any point *P* of the slab,

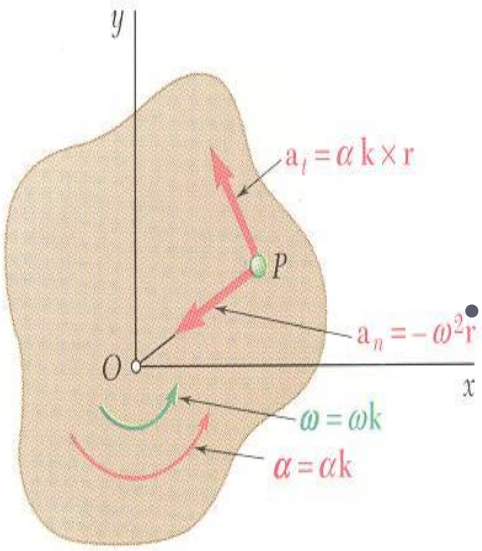
$$\vec{v} = \vec{\omega} \times \vec{r} = \omega \hat{k} \times \vec{r}$$

$$v = r\omega$$

- Acceleration of any point *P* of the slab,

$$\vec{a} = \vec{\alpha} \times \vec{r} + \vec{\omega} \times (\vec{\omega} \times \vec{r})$$

$$= \alpha \hat{k} \times \vec{r} - \omega^2 \vec{r}$$

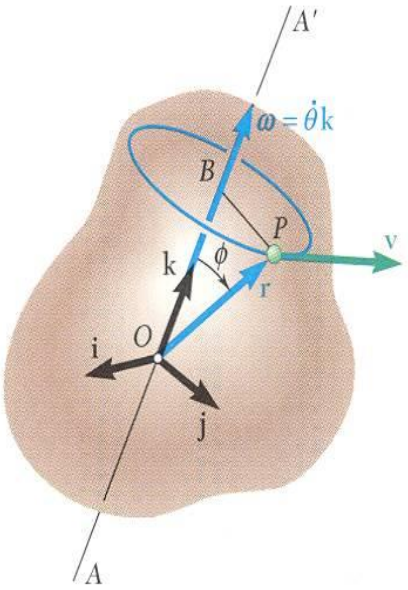


- Resolving the acceleration into tangential and normal components,

$$\vec{a}_t = \alpha \hat{k} \times \vec{r} \qquad a_t = r\alpha$$

$$\vec{a}_n = -\omega^2 \vec{r} \qquad a_n = r\omega^2$$

Equations Defining the Rotation of a Rigid Body About a Fixed Axis



- Motion of a rigid body rotating around a fixed axis is often specified by the type of angular acceleration.

- Recall $\omega = \dfrac{d\theta}{dt}$ or $dt = \dfrac{d\theta}{\omega}$

$$\alpha = \frac{d\omega}{dt} = \frac{d^2\theta}{dt^2} = \omega\frac{d\omega}{d\theta}$$

- *Uniform Rotation,* $\alpha = 0$:

$$\theta = \theta_0 + \omega t$$

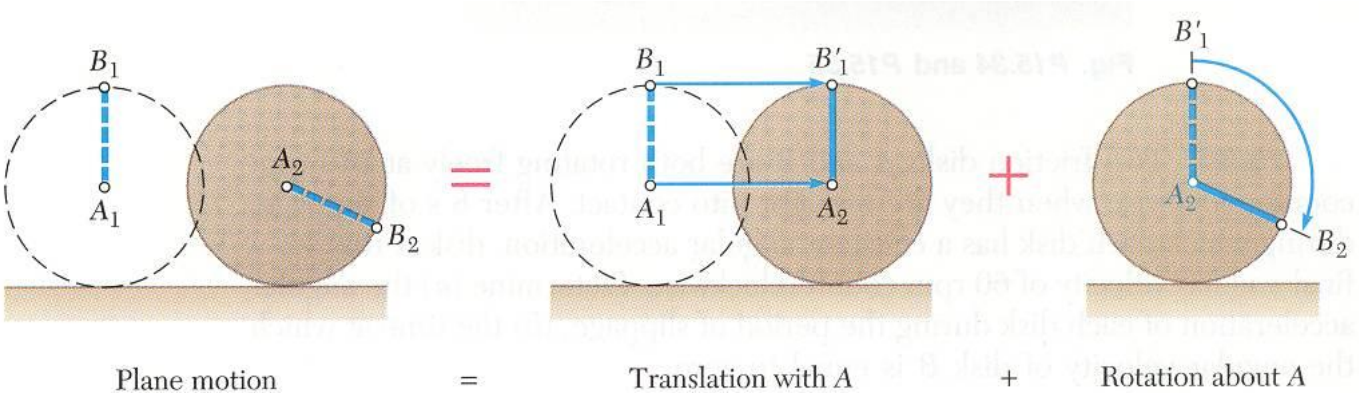
- *Uniformly Accelerated Rotation,* $\alpha =$ constant:

$$\omega = \omega_0 + \alpha t$$

$$\theta = \theta_0 + \omega_0 t + \tfrac{1}{2}\alpha t^2$$

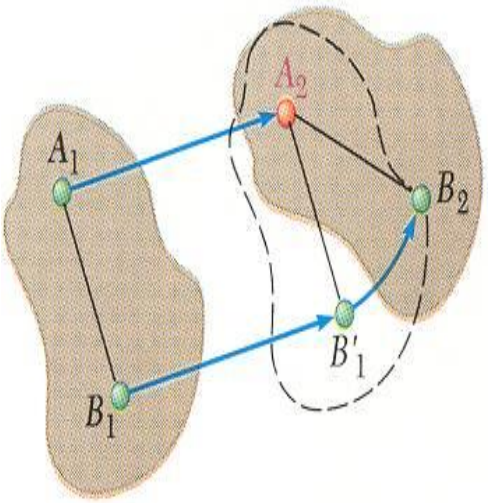
$$\omega^2 = \omega_0^2 + 2\alpha(\theta - \theta_0)$$

General Plane Motion



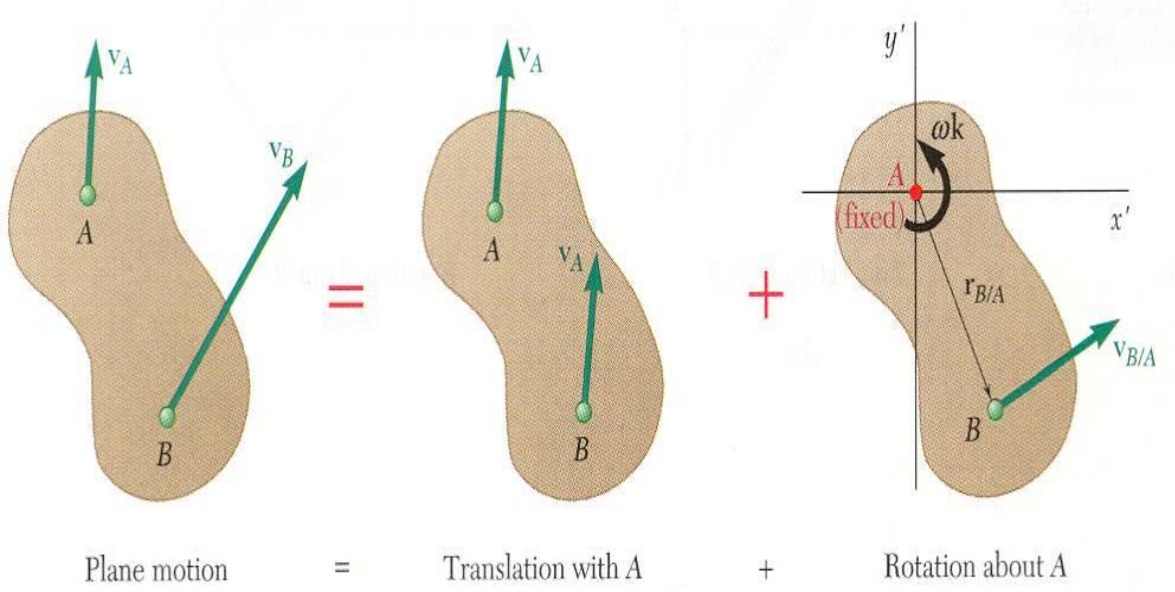
- *General plane motion* is neither a translation nor a rotation.

- General plane motion can be considered as the *sum* of a translation and rotation.

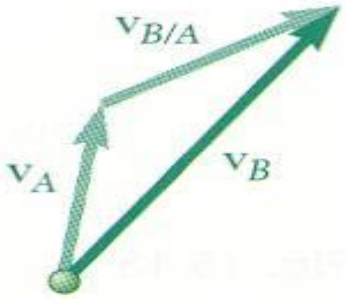


- Displacement of particles A and B to A_2 and B_2 can be divided into two parts:
  - translation to A_2 and B'_1
  - rotation of B'_1 about A_2 to B_2

Absolute and Relative Velocity in Plane Motion



- Any plane motion can be replaced by a translation of an arbitrary reference point A and a simultaneous rotation about A.



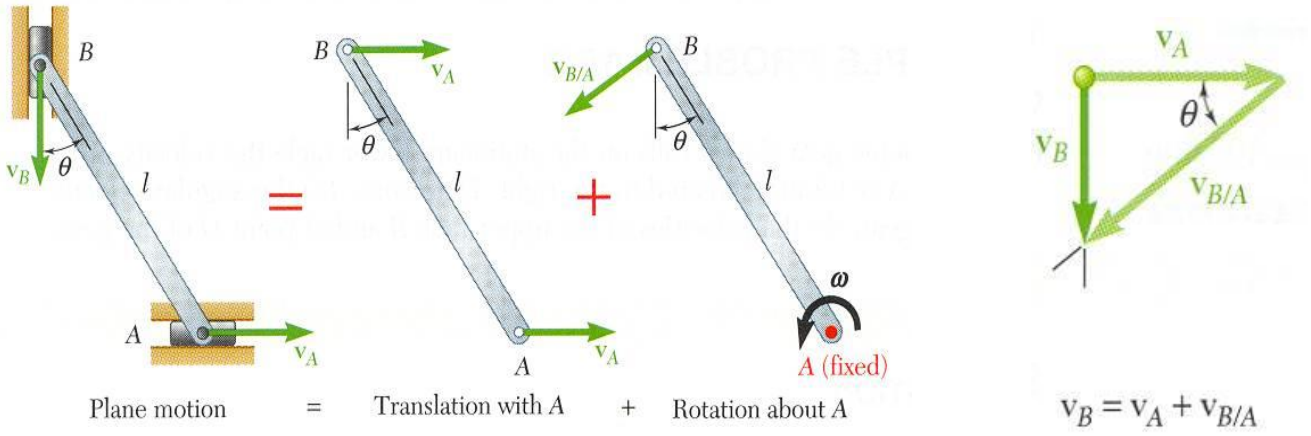
$$\mathbf{v}_B = \mathbf{v}_A + \mathbf{v}_{B/A}$$

$$\vec{v}_B = \vec{v}_A + \vec{v}_{B/A}$$

$$\vec{v}_{B/A} = \omega\vec{k} \times \vec{r}_{B/A} \qquad v_{B/A} = r\omega$$

$$\vec{v}_B = \vec{v}_A + \omega\vec{k} \times \vec{r}_{B/A}$$

Absolute and Relative Velocity in Plane Motion

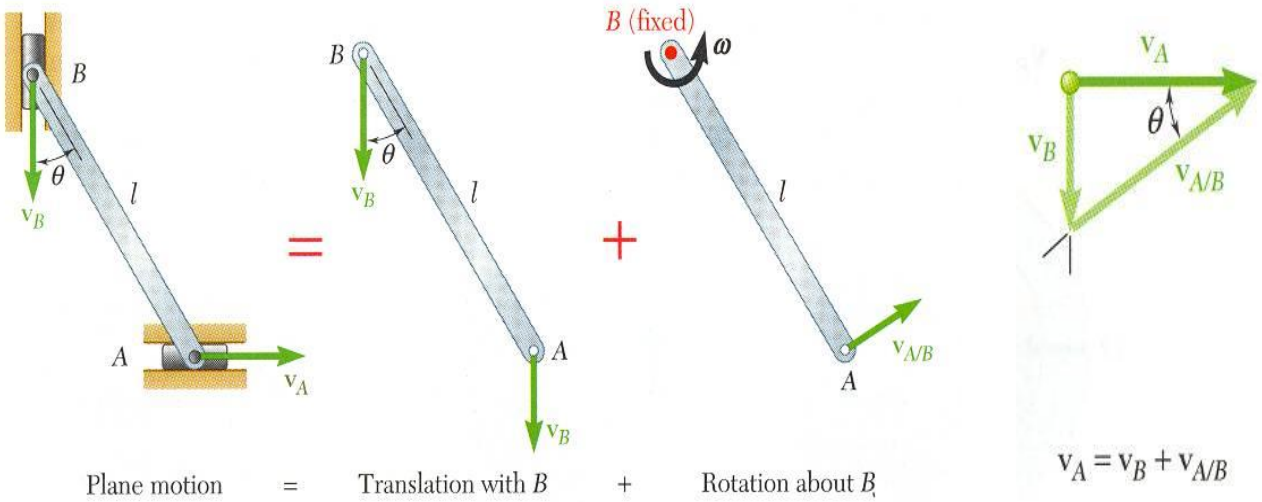


- Assuming that the velocity v_A of end A is known, wish to determine the velocity v_B of end B and the angular velocity ω in terms of v_A, l, and θ.

- The direction of v_B and $v_{B/A}$ are known. Complete the velocity diagram.

$$\frac{v_B}{v_A} = \tan\theta \qquad \frac{v_A}{v_{B/A}} = \frac{v_A}{l\omega} = \cos\theta$$

$$v_B = v_A \tan\theta \qquad \omega = \frac{v_A}{l\cos\theta}$$



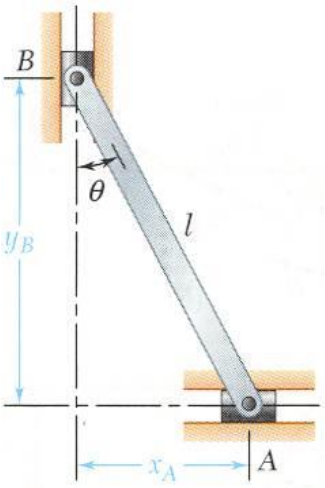
- Selecting point B as the reference point and solving for the velocity v_A of end A and the angular velocity ω leads to an equivalent velocity triangle.

- $v_{A/B}$ has the same magnitude but opposite sense of $v_{B/A}$. The sense of the relative velocity is dependent on the choice of reference point.

- Angular velocity ω of the rod in its rotation about B is the same as its rotation about A. Angular velocity is not dependent on the choice of reference point.

Analysis of Plane Motion in Terms of a Parameter

- In some cases, it is advantageous to determine the absolute velocity and acceleration of a mechanism directly.



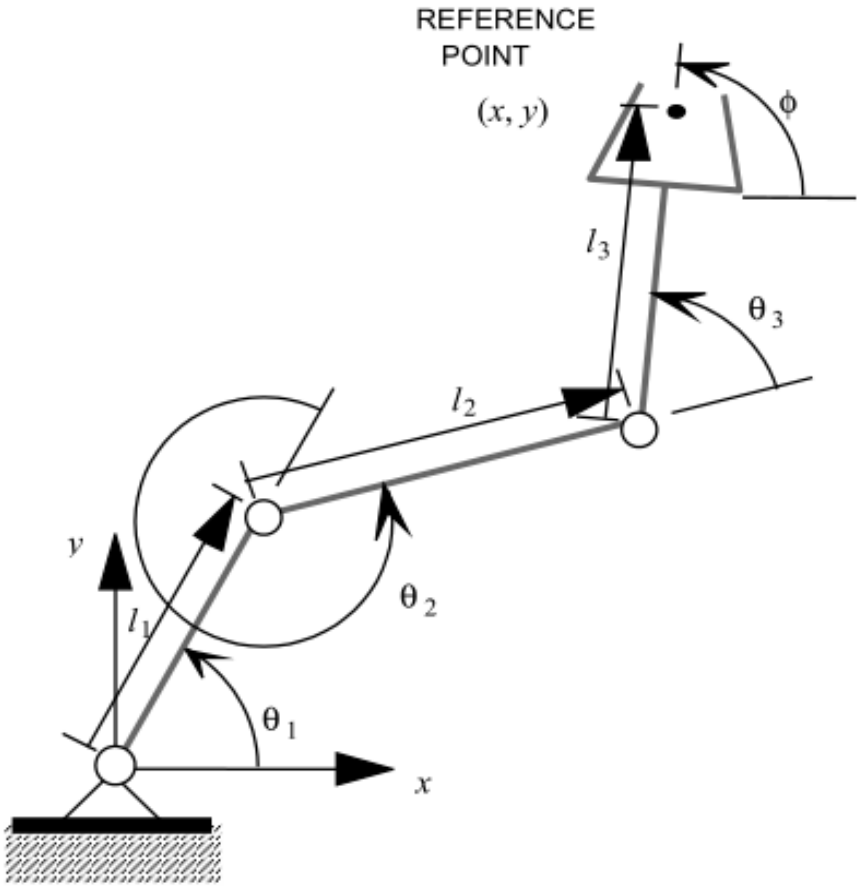
$$x_A = l\sin\theta \qquad y_B = l\cos\theta$$

$$v_A = \dot{x}_A \qquad v_B = \dot{y}_B$$
$$= l\dot{\theta}\cos\theta \qquad = -l\dot{\theta}\sin\theta$$
$$= l\omega\cos\theta \qquad = -l\omega\sin\theta$$

$$a_A = \ddot{x}_A \qquad a_B = \ddot{y}_B$$
$$= -l\dot{\theta}^2\sin\theta + l\ddot{\theta}\cos\theta \qquad = -l\dot{\theta}^2\cos\theta - l\ddot{\theta}\sin\theta$$
$$= -l\omega^2\sin\theta + l\alpha\cos\theta \qquad = -l\omega^2\cos\theta - l\alpha\sin\theta$$

Planar Robot

With a convenient choice of a reference point on the end effector, we can describe the position of the end effector using the coordinates of the reference point (x , y) and the orientation using the angle φ.
The three end effector coordinates (x , y , φ) completely specify the position and orientation of the end effector



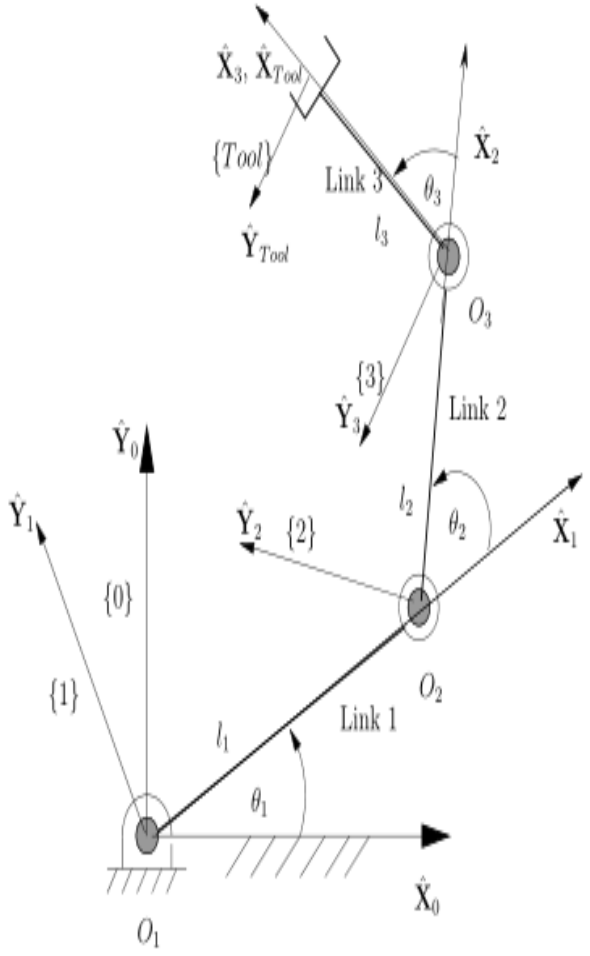
The joint variables and link lengths for a $3R$ planar manipulator

Example

A planar 3R manipulator

- ϕ represents orientation of the tool.
- From ${}^{0}_{Tool}[T]$

$$
\begin{aligned}
x &= l_1 c_1 + l_2 c_{12} + l_3 c_{123} \\
y &= l_1 s_1 + l_2 s_{12} + l_3 s_{123} \\
\phi &= \theta_1 + \theta_2 + \theta_3
\end{aligned}
\qquad (2)
$$



The planar 3R manipulator

Direct and Inverse Kinematics of Robot

- Forward (Direct)Kinematics:
  to determine where the robot's hand is?
  (If all joint variables are known)

- Inverse Kinematics:
  to calculate what each joint variable is?
  (If we desire that the hand be
  located at a particular point)

ROBOTS AS MECHANISM

♦Linkage mechanism:

Multiple type robot have multiple DOF.

(3 Dimensional, open loop, chain mechanisms)

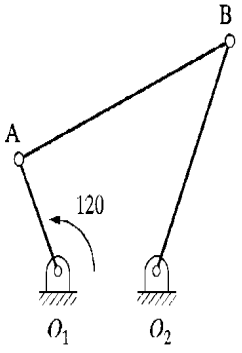


Fig. 2.1 A one-degree-of-freedom closed-loop four-bar mechanism

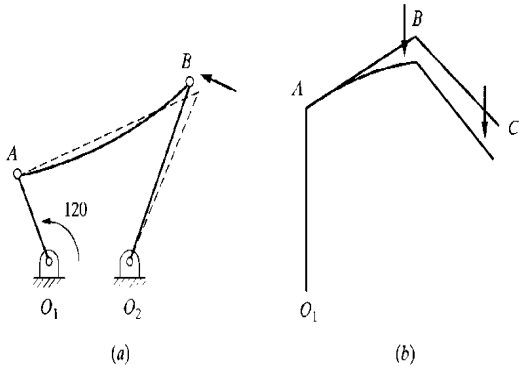


Fig. 2.2 (a) Closed-loop versus (b) open-loop mechanism

MATRIX REPRESENTATION

1-Representation of a Point in Space

♦A point *P* in space :

3 coordinates relative to a reference frame

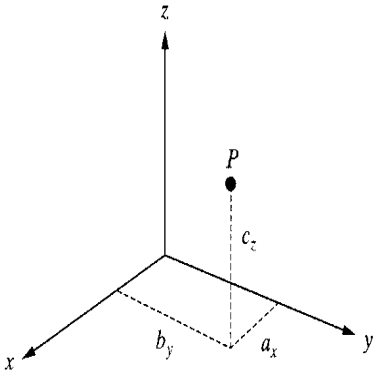


Fig. 2.3 Representation of point in space

$$P = a_x \hat{i} + b_y \hat{j} + c_z \hat{k}$$

2- Representation of a Vector in Space

- A Vector *P* in space :
  3 coordinates of its tail and of its head

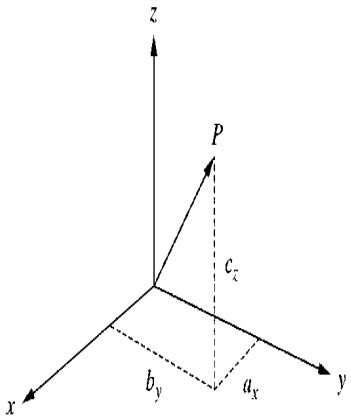


Fig. 2.4 Representation of a vector in space

$$\bar{P} = a_x \hat{i} + b_y \hat{j} + c_z \hat{k}$$

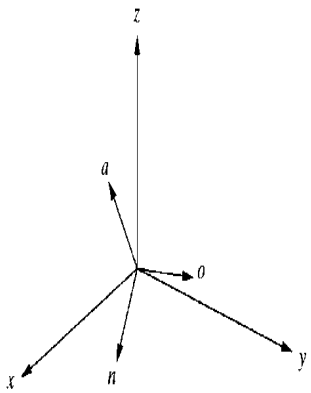
$$\bar{P} = \begin{bmatrix} x \\ y \\ z \\ w \end{bmatrix}$$

3- Representation of a Frame at the Origin of a Fixed-Reference Frame

- Each Unit Vector is mutually perpendicular. :
  normal, orientation, approach vector

Fig. 2.5 Representation of a frame at the origin of the reference frame

$$F = \begin{bmatrix} n_x & o_x & a_x \\ n_y & o_y & a_y \\ n_z & o_z & a_z \end{bmatrix}$$



3D Point

- A 3D point *P* is represented in homogeneous coordinates by a 4-dim. vector:

$$P = \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

- Note, that $p = \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix} \equiv \begin{bmatrix} \alpha x \\ \alpha y \\ \alpha z \\ \alpha \end{bmatrix}$

4- Representation of a Frame in a Fixed Reference Frame

♦Each Unit Vector is mutually perpendicular. :
normal, orientation, approach vector

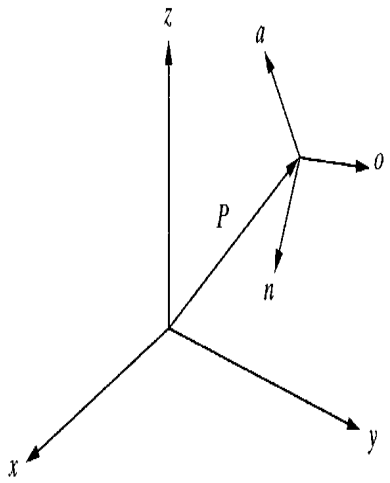


Fig. 2.6 Representation c
frame in a frame

$$F = \begin{bmatrix} n_x & o_x & a_x & P_x \\ n_y & o_y & a_y & P_y \\ n_z & o_z & a_z & P_z \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

5- Representation of a Rigid Body

♦An object can be represented in space by attaching a frame
to it and representing the frame in space.

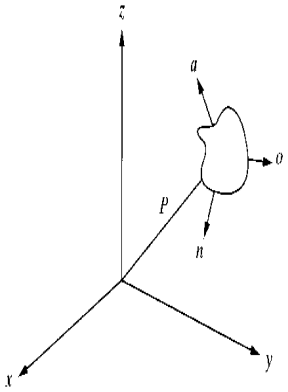
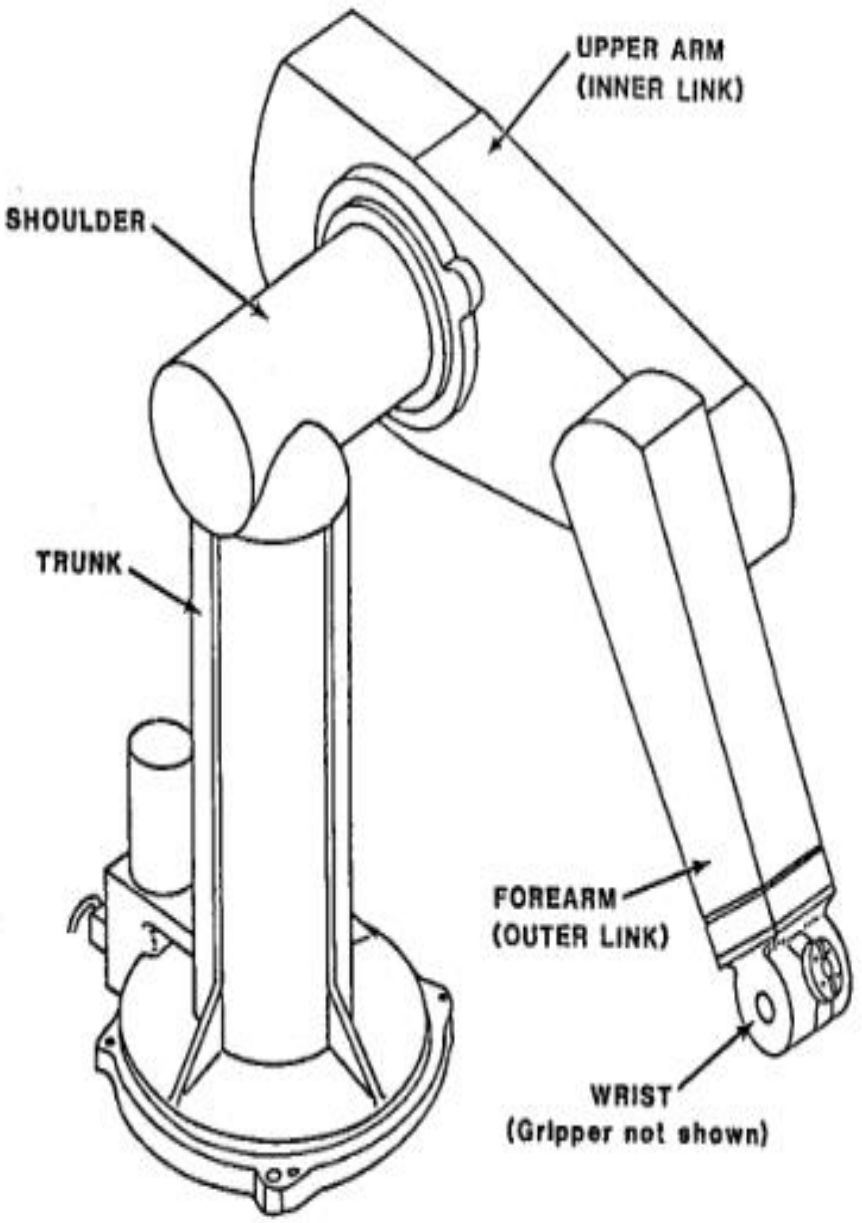


Fig. 2.8 Representation of object in space

$$F_{object} = \begin{bmatrix} n_x & o_x & a_x & P_x \\ n_y & o_y & a_y & P_y \\ n_z & o_z & a_z & P_z \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

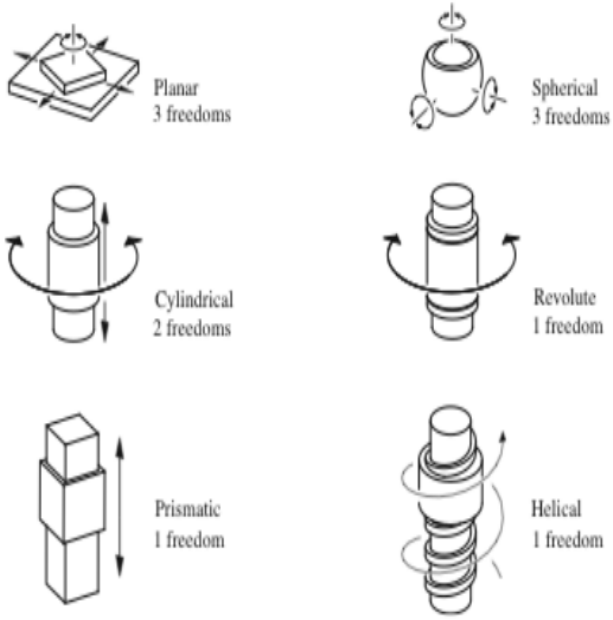
Arm Control to Reach:

- Mechanism design using:
- *Forward kinematics*
- Inverse kinematics



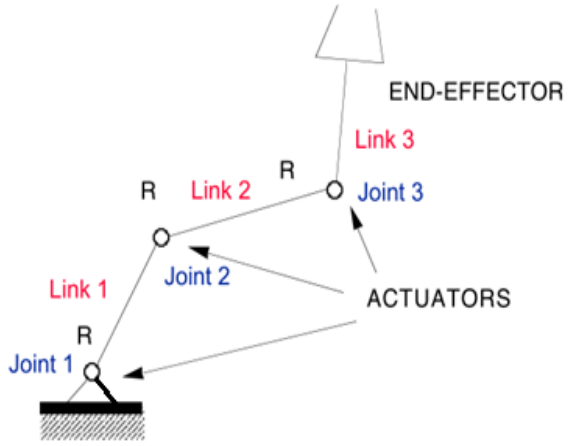
Kinematic Mechanisms

Link: rigid body
Joint: constraint
on two links
Kinematic mechanism:
links and joints



The Planar 3-R manipulator

- Planar kinematic chain
- All joints are revolute

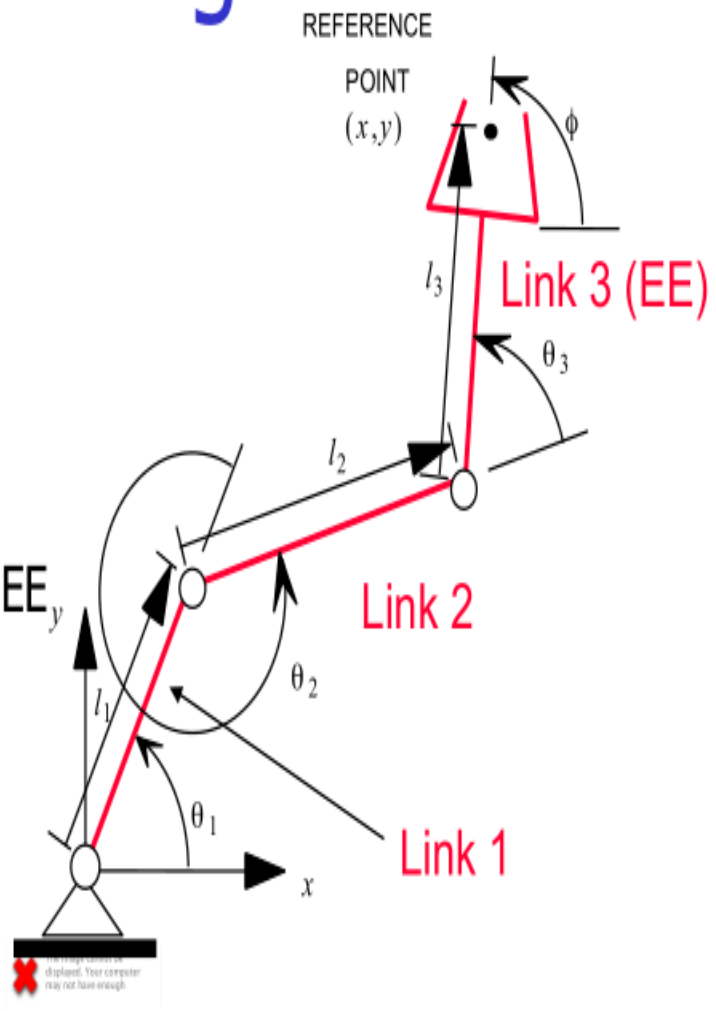


Kinematic modeling

- Link
- Actuated joint
- End effector (EE)
  - Reference point on EE
- Joint coordinates

  $\theta_1, \theta_2, \theta_3$
- End effector coordinates

  x, y, ϕ
- Link lengths (l_i)



Kinematic transformations

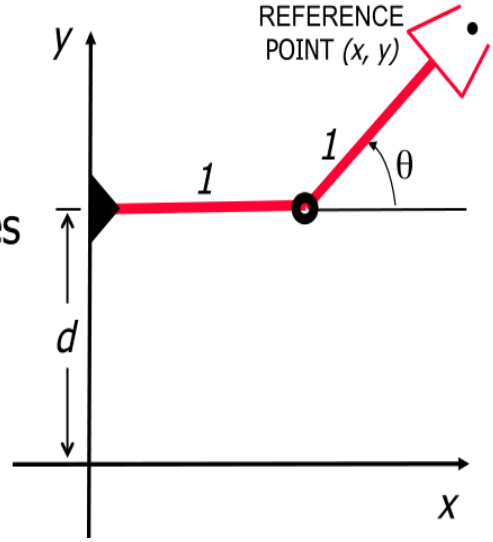
- Direct kinematics
  - Joint coordinates to end effector coordinates
    - Sensors are located at the joints. DK algorithm is used to figure out where the robot is in 3-D space.
    - Robot "thinks" in joint coordinates. Programmer/ engineer thinks in "world coordinates" or end effector coordinates.
- Inverse kinematics
  - End effector coordinates to joint coordinates
    - Given a desired position and orientation of the EE, we want to be able to get the robot to move to the desired goal. IK algorithm used to obtain the joint coordinates.
    - Essential for control.

Forward Kinematics

- Given mechanism description and joint values, express end effector pose in Cartesian coordinates
  - Example: two-link arm with one sliding, one rotating joint
- Configuration variables:
  - Joint coordinates d, θ
  - Link lengths (both 1)
- End effector coordinates
  - "Reference point" (x, y)
- Challenge: express as

  $x = x(d, \theta) =$

  $y = y(d, \theta) =$



Inverse Kinematics

- Given end effector pose in Cartesian coordinates, identify the joint values that yield specified pose
- Challenge: solve for joint values in terms of pose

  $\theta = \theta(x, y)$

  $=$

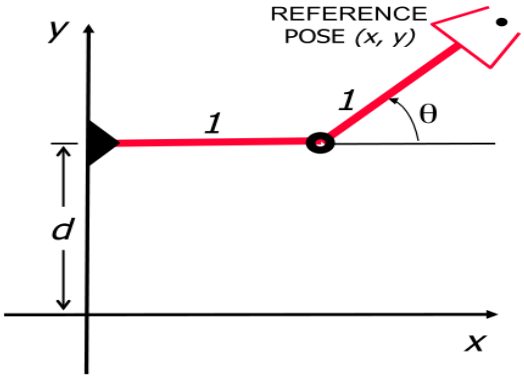
  $d = d(x, y)$

  $=$

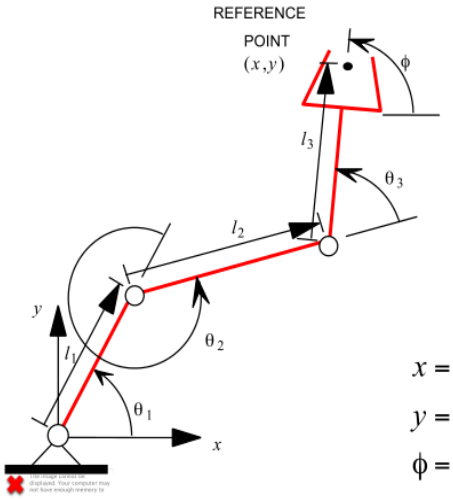
  Hints:

  $)^2$

  1



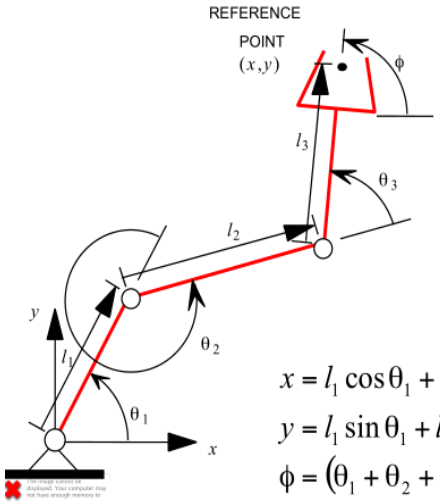
Direct kinematics



- Transform joint coordinates to end effector coordinates

$$x = l_1 \cos\theta_1 + l_2 \cos(\theta_1 + \theta_2) + l_3 \cos(\theta_1 + \theta_2 + \theta_3)$$
$$y = l_1 \sin\theta_1 + l_2 \sin(\theta_1 + \theta_2) + l_3 \sin(\theta_1 + \theta_2 + \theta_3)$$
$$\phi = (\theta_1 + \theta_2 + \theta_3)$$

Inverse kinematics



- Transform end effector coordinates to joint coordinates

$$x = l_1 \cos\theta_1 + l_2 \cos(\theta_1 + \theta_2) + l_3 \cos(\theta_1 + \theta_2 + \theta_3)$$
$$y = l_1 \sin\theta_1 + l_2 \sin(\theta_1 + \theta_2) + l_3 \sin(\theta_1 + \theta_2 + \theta_3)$$
$$\phi = (\theta_1 + \theta_2 + \theta_3)$$

Given x, y, ϕ, solve for θ_1, θ_2, θ_3

A Trigonometric Digression

$$\sin\theta = -\sin(-\theta\ ) = -\cos(\theta + \frac{\pi}{2}) = \cos(\theta - \frac{\pi}{2})$$

$$\cos\theta = \cos(-\theta\ ) = \sin(\theta + \frac{\pi}{2}) = -\sin(\theta - \frac{\pi}{2})$$

$$\cos^2\theta + \sin^2\theta = 1$$

$$\sin\theta = \pm\sqrt{1 - \cos^2\theta}$$

$$\sin(\alpha + \beta\ ) = \sin\alpha\ \ \cos\beta + \cos\alpha\ \ \sin\beta$$
$$\cos(\alpha + \beta\ ) = \cos\alpha\ \ \cos\beta - \sin\alpha\ \ \sin\beta$$
$$\sin(\alpha - \beta\ ) = \sin\alpha\ \ \cos\beta - \cos\alpha\ \ \sin\beta$$
$$\cos(\alpha - \beta\ ) = \cos\alpha\ \ \cos\beta + \sin\alpha\ \ \sin\beta$$

$\cos\theta = (1-t^2) / (1+t^2)$

$\sin\theta = 2t / (1+t^2)$

Arctangents

By definition $\tan\theta = \dfrac{\sin\theta}{\cos\theta}$ **, but tan has a period of π, whereas sin and cos have a period of 2π. If it is known that**

$$x = r\cos\theta,\ \ y = r\sin\theta,\ \ r > 0$$

then recover with

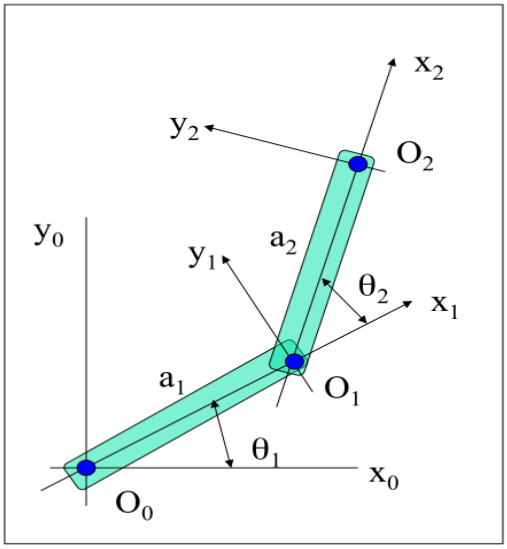
$$\theta = \arctan\left(\frac{y}{x}\right) = \arctan\left(\frac{r\sin\theta}{r\cos\theta}\right)$$

But, this only gives solution in first and fourth quadrants. The signs of x and y actually uniquely determine the quadrant, so use:

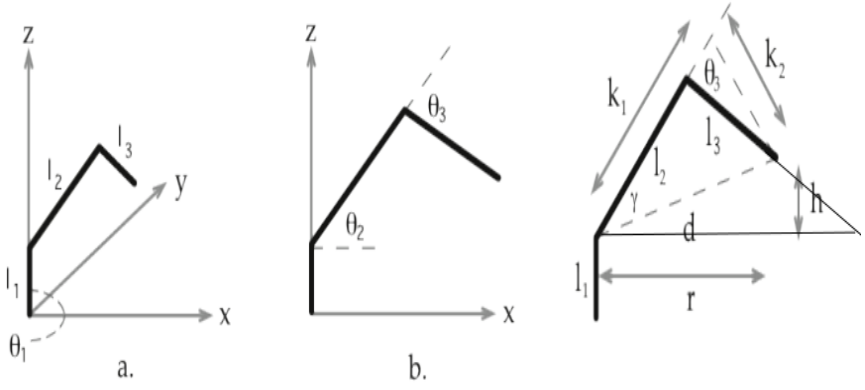
$$\theta \leftarrow \text{atan2}(y,x)$$

and then $\tan\theta = \dfrac{y}{x}$, $\sin\theta = \dfrac{y}{\sqrt{x^2 + y^2}}$, $\cos\theta = \dfrac{x}{\sqrt{x^2 + y^2}}$

Planar Example



Example: 3DOF Revolute Arm



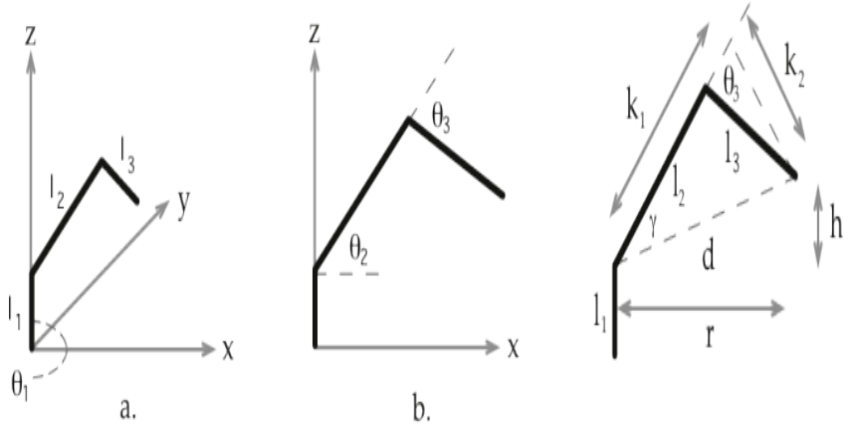
Three revolute joints. Write s_1 and c_1 for $\sin\theta_1$ and $\cos\theta_1$ respectively.

Downward elbow angle is $\theta_{3-2} = \theta_3 - \theta_2$ write:

$$s_{3-2} = \sin\theta_{3-2} = \sin(\theta_3 - \theta_2)$$

$$c_{3-2} = \cos\theta_{3-2} = \cos(\theta_3 - \theta_2)$$

Forward Kinematics



$$r = \sqrt{x^2 + y^2} = l_2 c_2 + l_3 c_{3-2}$$

$$x = [l_2 c_2 + l_3 c_{3-2}]c_1$$

$$y = [l_2 c_2 + l_3 c_{3-2}]s_1$$

$$h = z - l_1 = l_2 s_2 - l_3 s_{3-2}$$

$$z = l_1 + l_2 s_2 - l_3 s_{3-2}$$

Solution for Planar Example

$$x^2 + y^2 = a_1^2 + a_2^2 - 2a_1 a_2 \cos(\pi - \theta_2)$$

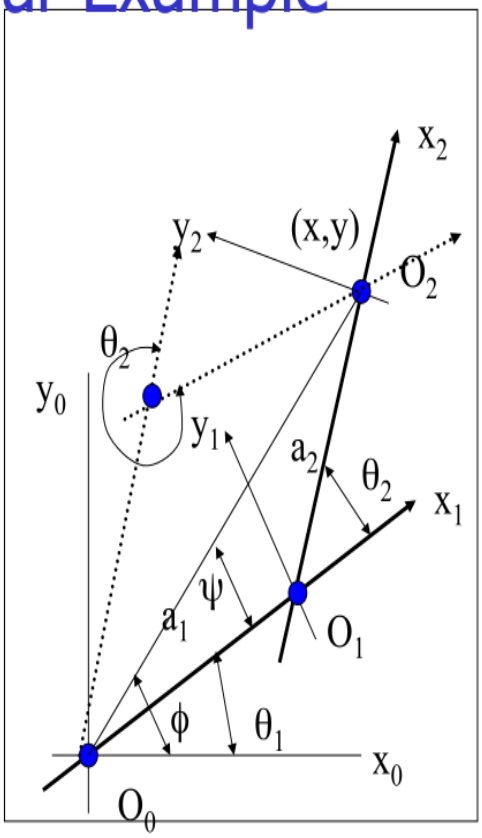
$$\cos\theta_2 = \frac{x^2 + y^2 - a_1^2 - a_2^2}{2a_1 a_2}$$

for greater accuracy

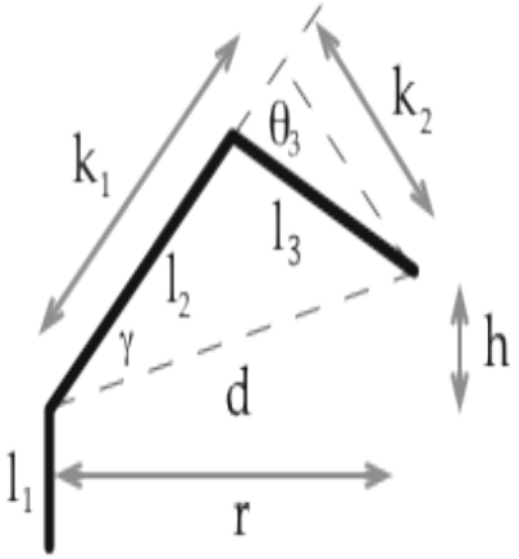
$$\tan^2\frac{\theta_2}{2} = \frac{1-\cos\theta}{1+\cos\theta} = \frac{2a_1a_2 - x^2 - y^2 + a_1^2 + a_2^2}{2a_1a_2 + x^2 + y^2 - a_1^2 - a_2^2}$$

$$= \frac{\left(a_1^2 + a_2^2\right)^2 - \left(x^2 + y^2\right)}{\left(x^2 + y^2\right) - \left(a_1^2 - a_2^2\right)^2}$$

$$\theta_2 = \pm 2\tan^{-1}\sqrt{\frac{\left(a_1^2 + a_2^2\right)^2 - \left(x^2 + y^2\right)}{\left(x^2 + y^2\right) - \left(a_1^2 - a_2^2\right)^2}}$$



Inverse Kinematics of 3DOF Manipulator



$$r = \sqrt{x^2 + y^2} = l_2 c_2 + l_3 c_{3-2}$$

$$h = z - l_1 = l_2 s_2 - l_3 s_{3-2}$$

$$x^2 + y^2 + (z - l_1)^2 = r^2 + h^2$$

$$= l_2^2 c_2^2 + l_3^2 c_{3-2}^2 + 2 l_2 l_3 c_2 [c_3 c_2 + s_3 s_2] + l_2^2 s_2^2 + l_3^2 s_{3-2}^2 - 2 l_2 l_3 s_2 [s_3 c_2 - c_3 s_2]$$

$$= l_2^2 + l_3^2 + 2 l_2 l_3 [c_2^2 c_3 + s_2^2 c_3]$$

$$= l_2^2 + l_3^2 + 2 l_2 l_3 c_3$$

$$c_3 = \frac{x^2 + y^2 + (z - l_1)^2 - l_2^2 - l_3^2}{2 l_2 l_3}$$

Solving for the Elbow

Can choose an "up elbow" solution with $s_3 = +\sqrt{1 - c_3^2}$

or a "down elbow" solution with $s_3 = -\sqrt{1 - c_3^2}$

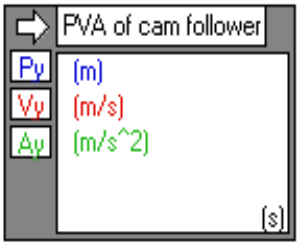
where $c_3 = \dfrac{x^2 + y^2 + (z - l_1)^2 - l_2^2 - l_3^2}{2 l_2 l_3}$

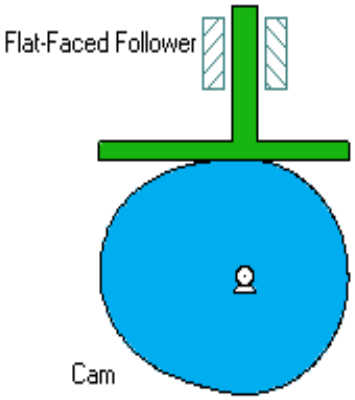
In either case can then recover θ_3 with

$$\theta_3 = \text{atan2}(s_3, c_3)$$

Cams

- Function generator
- Can generate a true dwell



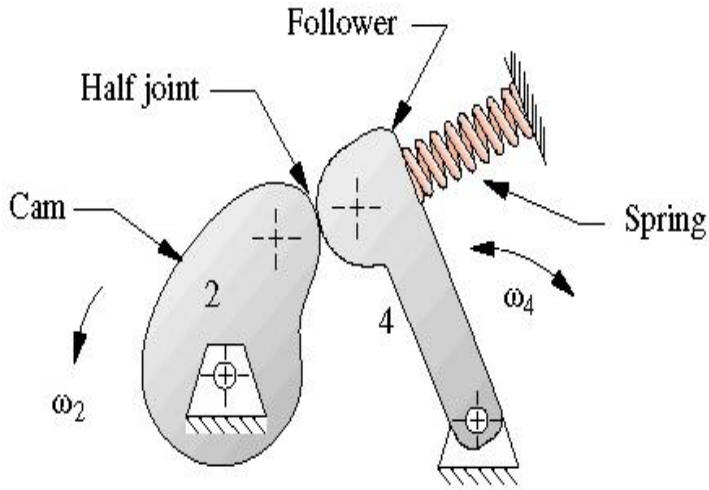


- **Cam Terminology**

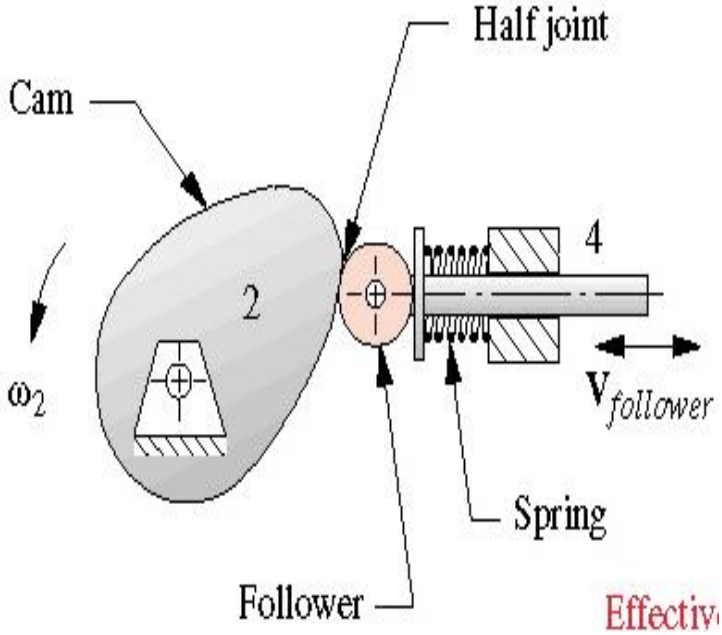
- Type of follower motion (rotation, translation)
- Type of joint closure (force, form)
- Type of follower (roller, mushroom, flat)
- Direction of follower motion (radial, axial)
- Type of motion constraints (critical extreme position(CEP) and critical path motion (CPM))
- Type of motion program (rise-fall (RF), rise-fall-dwell (RFD), rise-dwell-fall-dwell (RDFD)

Type of Follower Motion

Oscillating follower



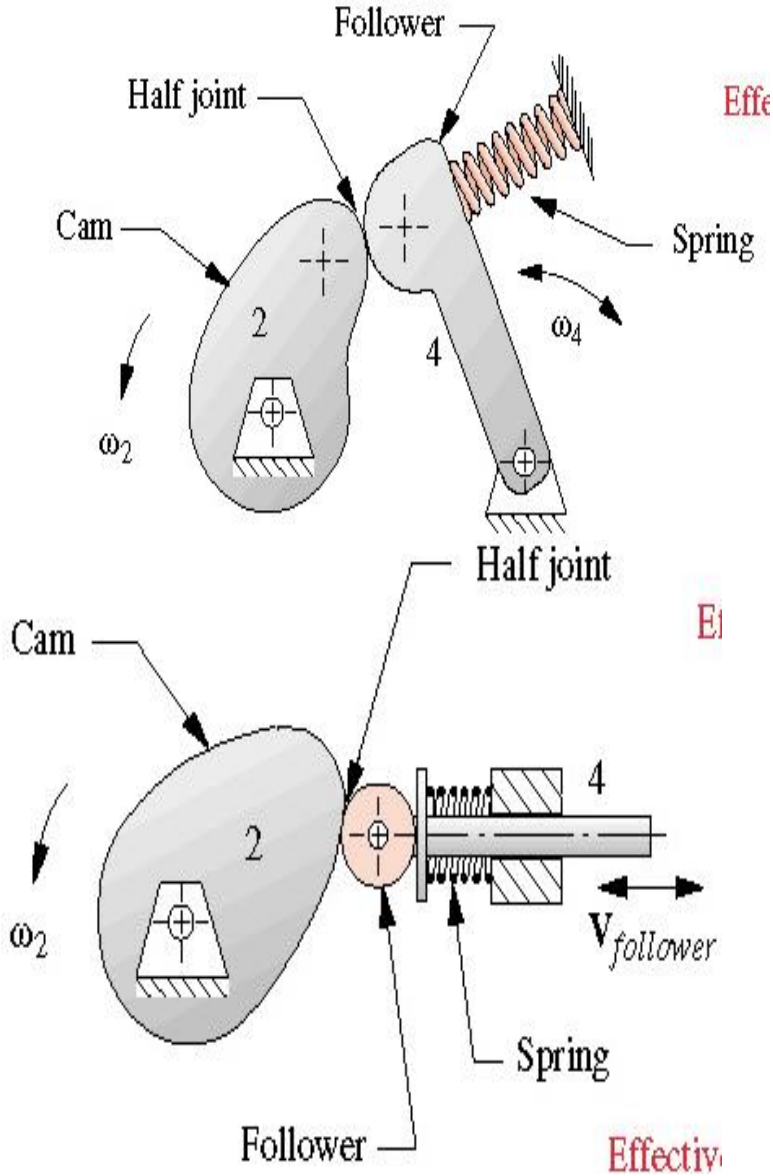
Translating follower



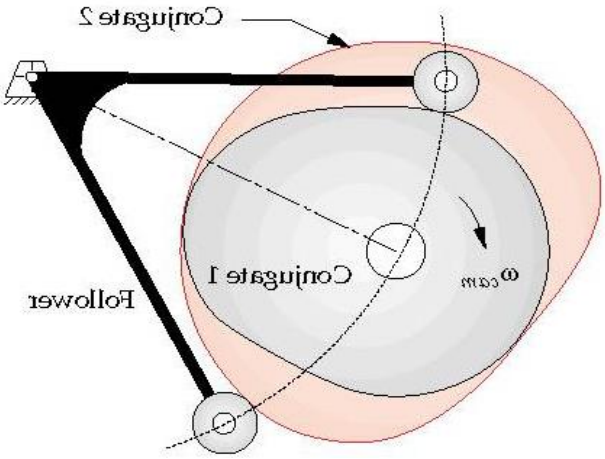
Type of Joint Closure

Force and **form** closed cams

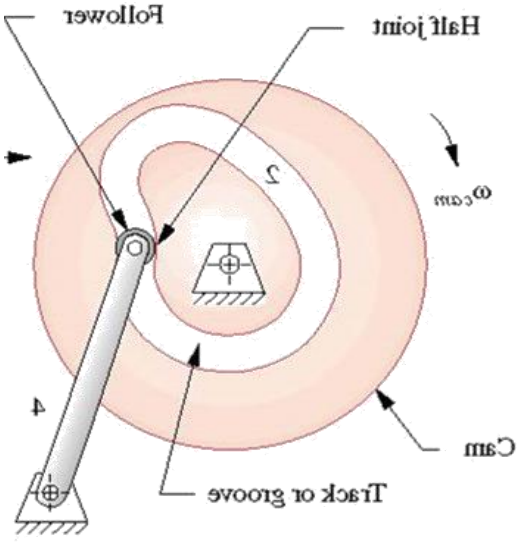
- **Force** closed cams require an external force to keep the cam in contact with the follower
- A **spring** usually provides this force

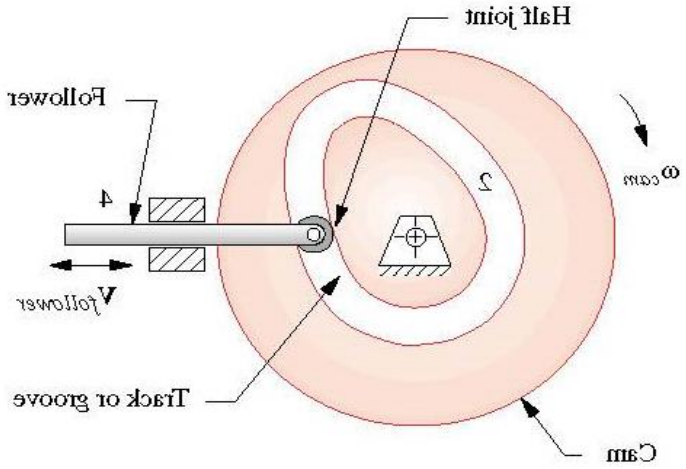


- **Form** closed cams are closed by joint geometry
- Slot milled out of the cam



Type of Joint Closure





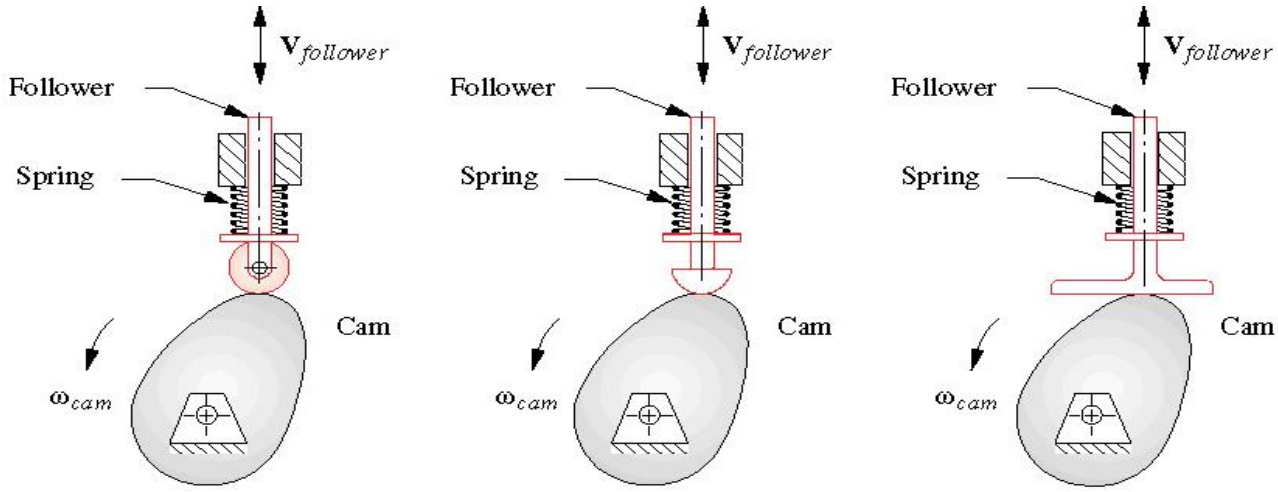
Types of Followers

Roller Follower

Mushroom Follower

Flat-Faced Follower

- Roller Follower
- Mushroom Follower
- Flat-Faced Follower



For kinematic analysis:

the follower displacement versus time curve is preferred. To assist in the task of designing a cam shape, the follower displacement versus cam angle curve is desired. Relating the cam rotation and time is a straightforward process using the theory presented the the following equation :

$$\omega_{cam} = \frac{\Delta\theta}{\Delta t}$$

Cam rotation during an interval of follower motion is typically expressed by the symbol β. Likewise, the time consumed during an interval is designated T. The amount of follower rise,or fall, during an interval is designated H. Rewriting the equation

using cam nomenclature, gives the relationship between cam rotation and time for an arbitrary interval, i:

$$\beta_i = (\omega_{cam})(T_i)$$

can also be used to determine the required speed of the cam, by observing the time consumed during one cycle.

$$\omega_{cam} = \frac{1\,\text{rev}}{\Sigma T}$$

where

ΣT_i = the total time for all motion intervals that comprise one cycle.

EXAMPLE 1

A cam is to be used for a platform that will repeatedly lift boxes from a lower conveyor to an upper conveyor. This machine is shown in Figure 9.5. Plot a displacement diagram and determine the required speed of the cam when the follower motion sequence is as follows:

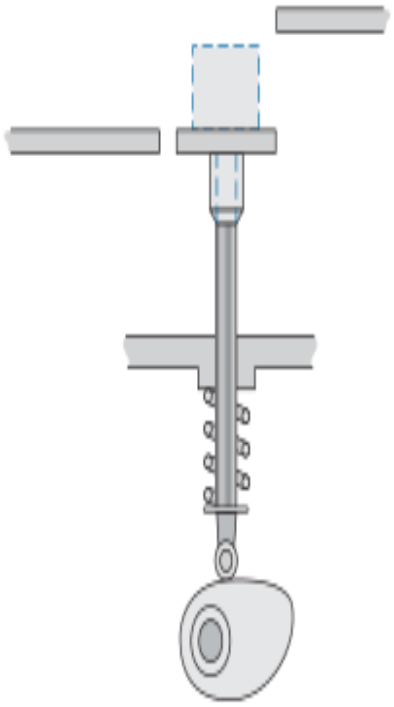


FIGURE 9.5 Cam system for Example Problem 9.1.

1. Rise 2 in. in 1.2 s.
2. Dwell for 0.3 s.
3. Fall 1 in. in 0.9 s.
4. Dwell 0.6 s.
5. Fall 1 in. in 0.9 s.

SOLUTION:

1. ***Calculate the Time for a Full Cycle***

The total time to complete the full cycle is needed to determine the required speed of the cam.

$$\Sigma T_i = T_1 + T_2 + T_3 + T_4 + T_5 = (1.2 + 0.3 + 0.9 + 0.6 + 0.9)\text{s} = 3.9\text{s}$$

2. ***Calculate the Required Rotational Speed of the Cam***

Then from equation (9.2),

$$\omega_{\text{cam}} = \frac{1\,\text{rev}}{\Sigma T_i} = \frac{1\,\text{rev}}{3.9\,\text{s}} = 0.256\,\text{rev/s}\left(\frac{60\,\text{s}}{1\,\text{min}}\right) = 15.38\ \text{rpm}$$

3. ***Determine the Cam Rotation for Each Follower Motion Interval***

The angular increment of the cam consumed by each follower motion sequence is determined by equation (9.1).

$$\beta_1 = (\omega_{\text{cam}})(T_i) = (0.256\,\text{rev/s})(1.2\,\text{s}) = 0.307\,\text{rev}$$
$$= (0.307\,\text{rev})(360°/1\ \text{rev}) = 110.5°$$

$$\beta_2 = (0.256\,\text{rev/s})(0.3\,\text{s}) = 0.077\,\text{rev} = 27.6°$$

$$\beta_3 = (0.256\,\text{rev/s})(0.9\,\text{s}) = 0.230\,\text{rev} = 82.9°$$

$$\beta_4 = (0.256\,\text{rev/s})(0.6\,\text{s}) = 0.154\,\text{rev} = 55.3°$$

$$\beta_5 = (0.256\,\text{rev/s})(0.9\,\text{s}) = 0.230\,\text{rev} = 82.9°$$

4. ***Plot the Displacement Diagram***

The resulting displacement diagram with both cam angle and time displayed on the horizontal axis is shown in Figure 9.6. Notice that a curved displacement profile was constructed during the rise and fall sequences. Dynamic considerations dictate the actual shape of the rise and fall sections.

4. *Plot the Displacement Diagram*

The resulting displacement diagram with both cam angle and time displayed on the horizontal axis is shown in Figure 9.6. Notice that a curved displacement profile was constructed during the rise and fall sequences. Dynamic considerations dictate the actual shape of the rise and fall sections.

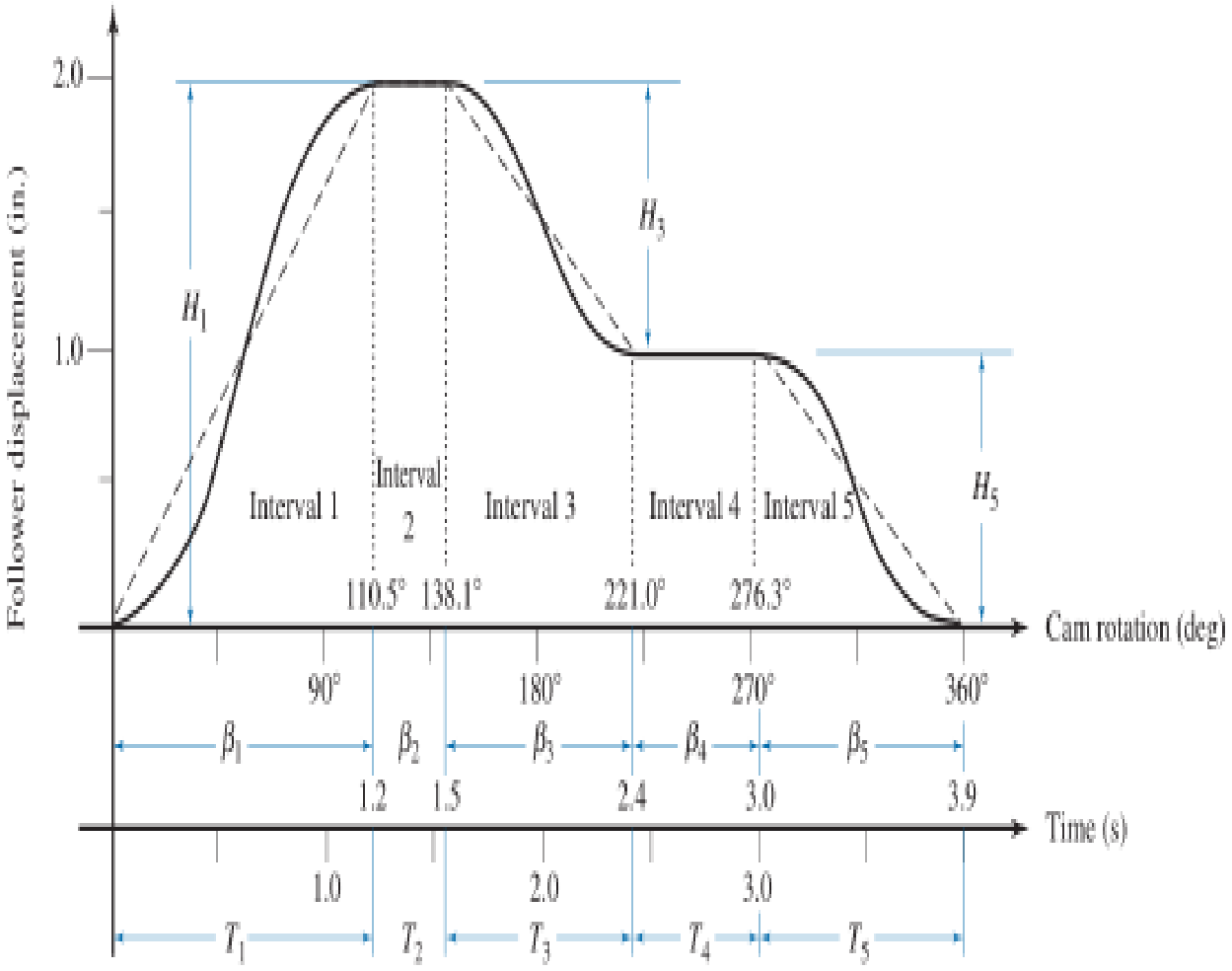


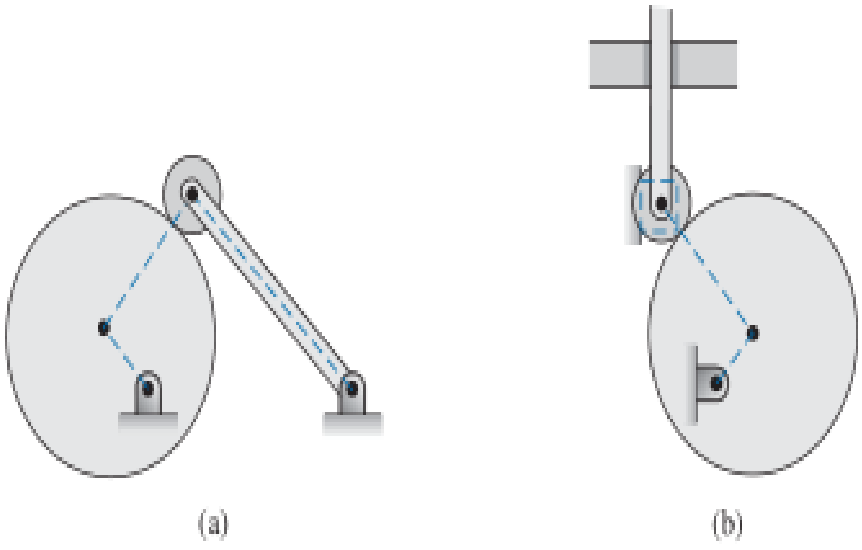
FIGURE 9.6 Displacement diagram for Example Problem

EQUIVALENT LINKAGES

Up to this point in the text, the examples of motion analysis involved only mechanisms with primary joints; that is, pin and sliding joints. Recall from Chapter 1 that a higher-order joint,such as a cam or gear joint,involves rolling and sliding
motion. Both cams and gears are the focus in later chapters.

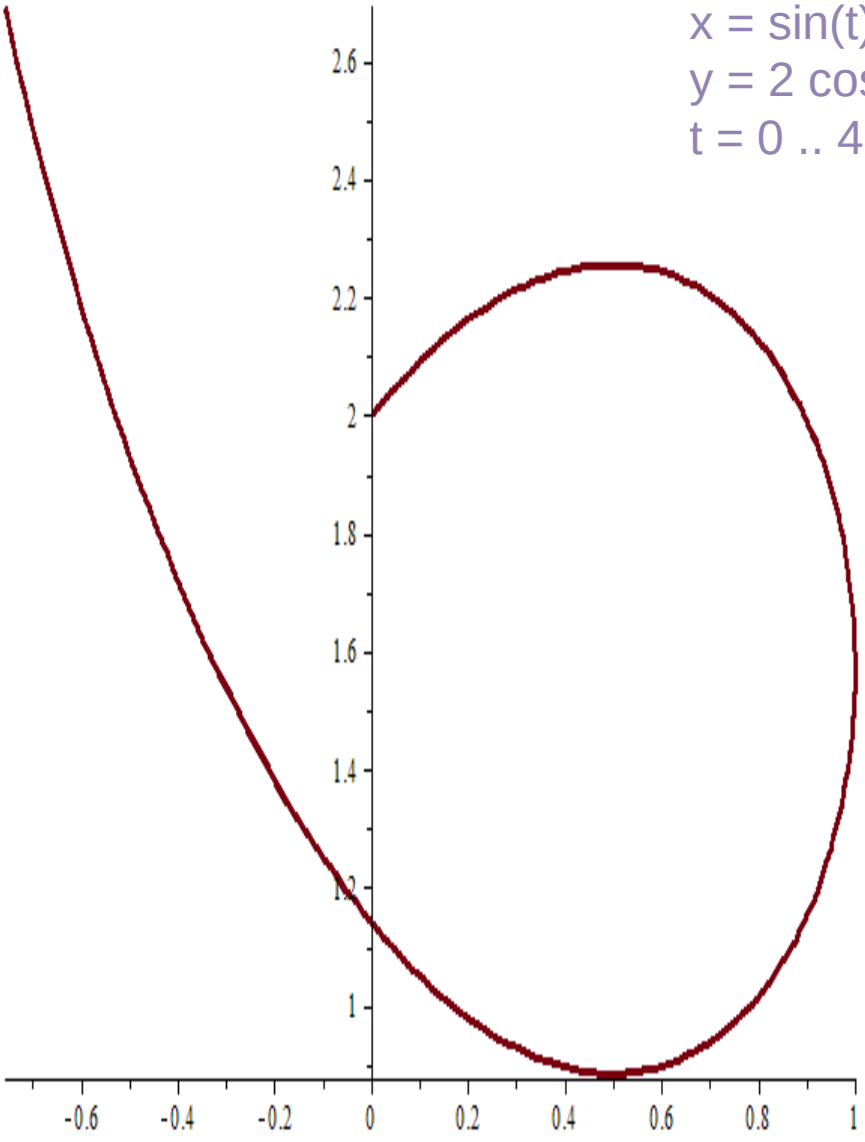
However, the motion analysis of mechanisms with higher order joints can be performed using the concepts already presented.

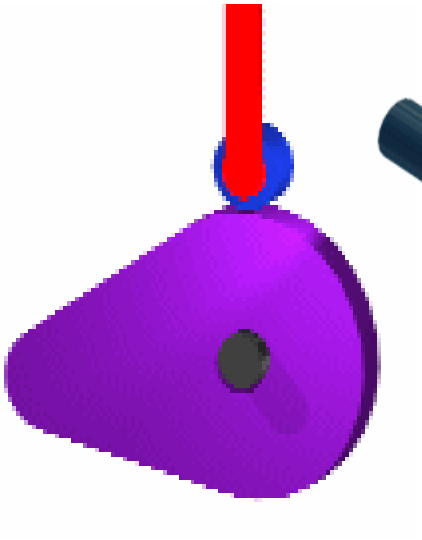
Velocity and acceleration analysis of mechanisms that utilize higher-order joints(e.g cams and gears) is greatly simplified by constructing an equivalent linkage. This method converts the instantaneous configuration of a mechanism to an equivalent linkage, where the links are connected by primary joints the fig. below illustrates two cam mechanisms

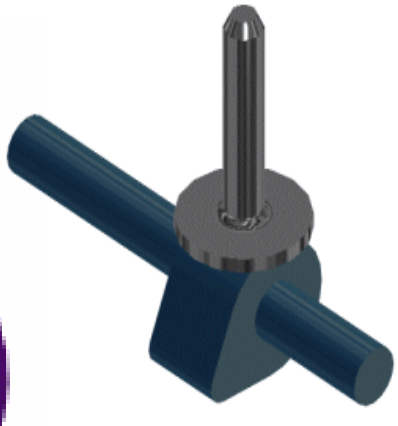


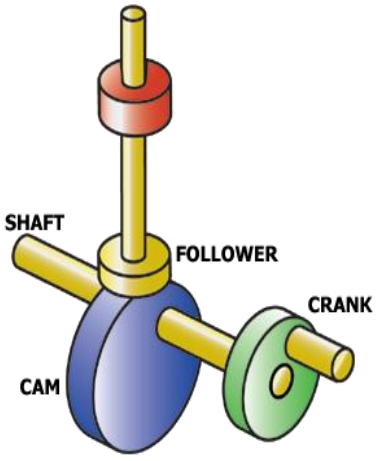
Equivalent linkages.

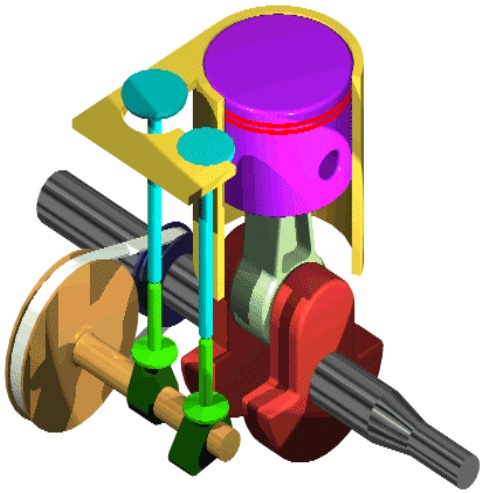
Parametric Equations

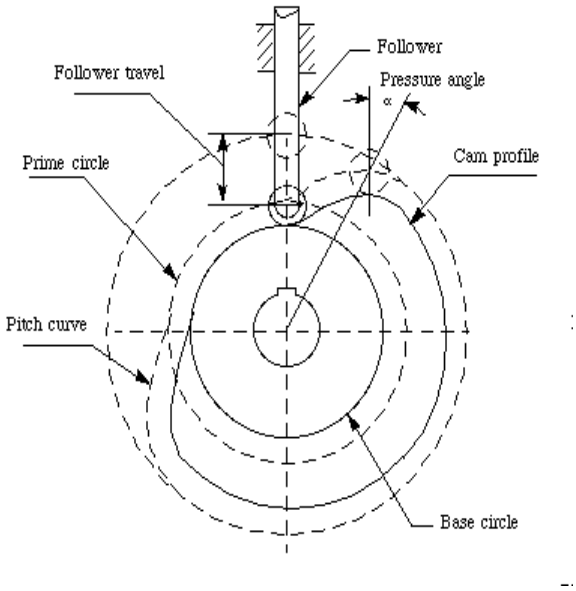




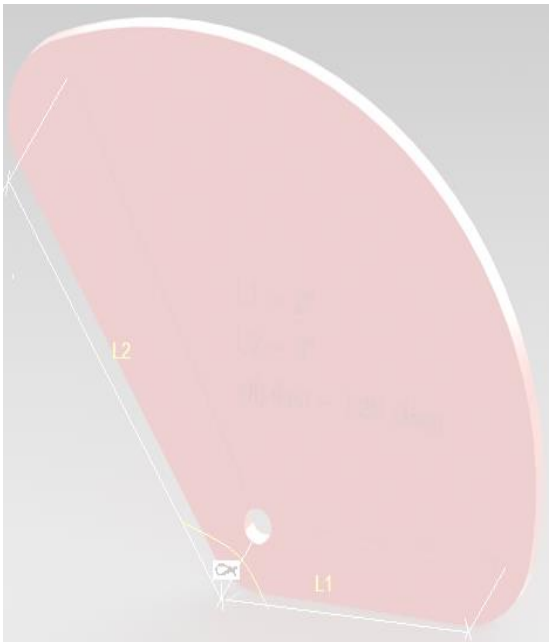


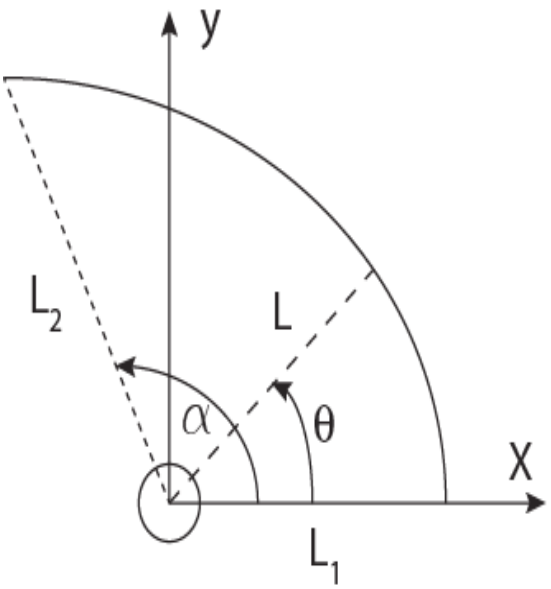

SHAFT
FOLLOWER
CRANK
CAM




Follower
Follower travel
Pressure angle
α
Prime circle
Cam profile
Pitch curve
Base circle

- Our Problem to:
- Design a disc cam (for use with a flat follower) such that:
  - follower height is L_1 when cam angle is 0°
  - follower height is L_2 when cam angle is α
  - the relationship between the height, L, and the cam angle, θ, is linear





We need to get the function of the cam profile and then draw a curve in SolidWorks that exactly models this profile.

Determine cam profile equation

- Would like to have y = f(x).
- We want a linear relationship between L and θ.

$$L = A\,\theta + B$$

Determine A and B.

- When θ = 0, L = L_1 ; when θ = α, L = L_2

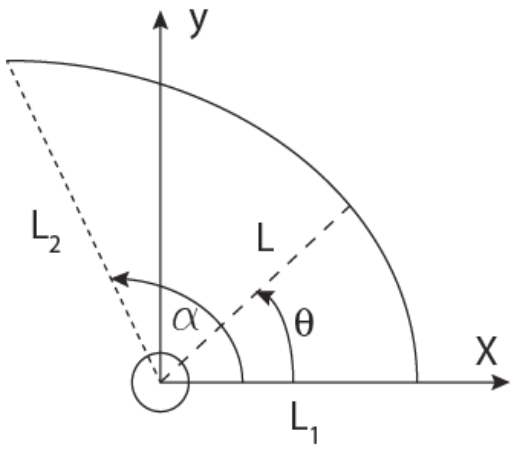
$L_1 = A\,(0) + B$

$L_2 = A\,(\alpha) + B$

$$B = L_1$$

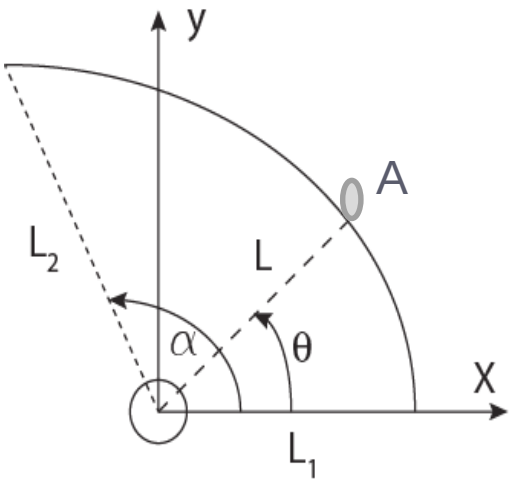
$$A = \frac{L_2 - L_1}{\alpha}$$

Cam profile equation



$$L = A\,\theta + B$$

$$L = \left(\frac{L_2 - L_1}{\alpha}\right)\theta + L_1$$



- Now we'll get the x and y coord of point A (an arbitrary point)

$$x_A = L\cos\theta$$

$$y_A = L\sin\theta$$

substitute for L

$$x = \left[\left(\frac{L_2 - L_1}{\alpha}\right)\theta + L_1\right]\cos\theta$$

$$y = \left[\left(\frac{L_2 - L_1}{\alpha}\right)\theta + L_1\right]\sin\theta$$

$$x = \left[\left(\frac{L_2 - L_1}{\alpha}\right)\theta + L_1\right]\cos\theta$$

$$y = \left[\left(\frac{L_2 - L_1}{\alpha}\right)\theta + L_1\right]\sin\theta$$

- Instead we have y and x as a function of θ.

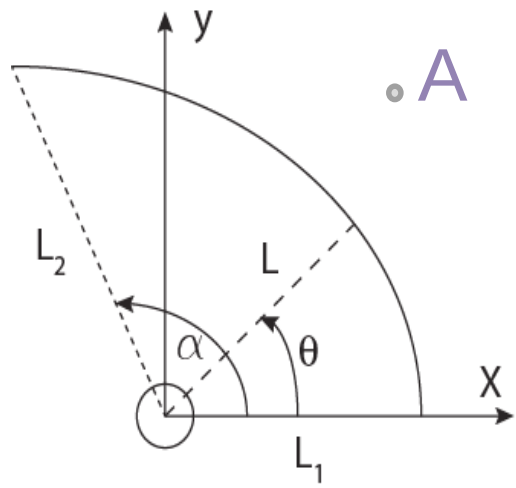
  This is called a parametric representation of x and y.

- Example:

  L_1 = 2" (when θ = 0)

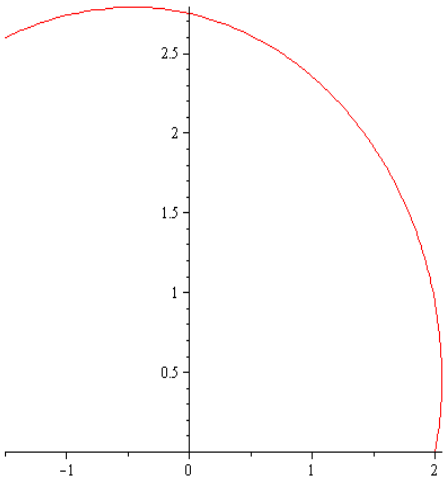
  L_2 = 3" corresponding to $\frac{2\pi}{3}$

α = (120°)



$$x = \left[\left(\frac{3}{2\pi}\right)\theta + 2\right]\cos\theta$$

$$y = \left[\left(\frac{3}{2\pi}\right)\theta + 2\right]\sin\theta$$

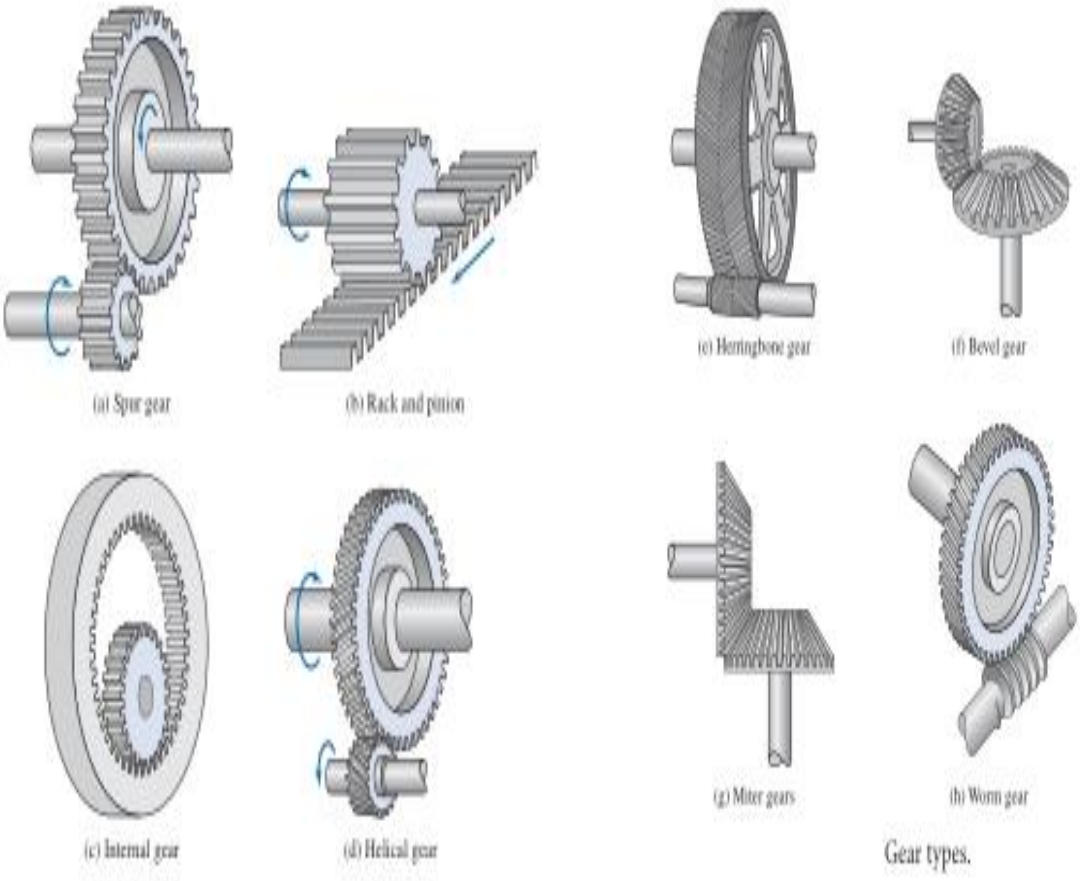


L_1 = 2"
L_2 = 3"
α = 120°

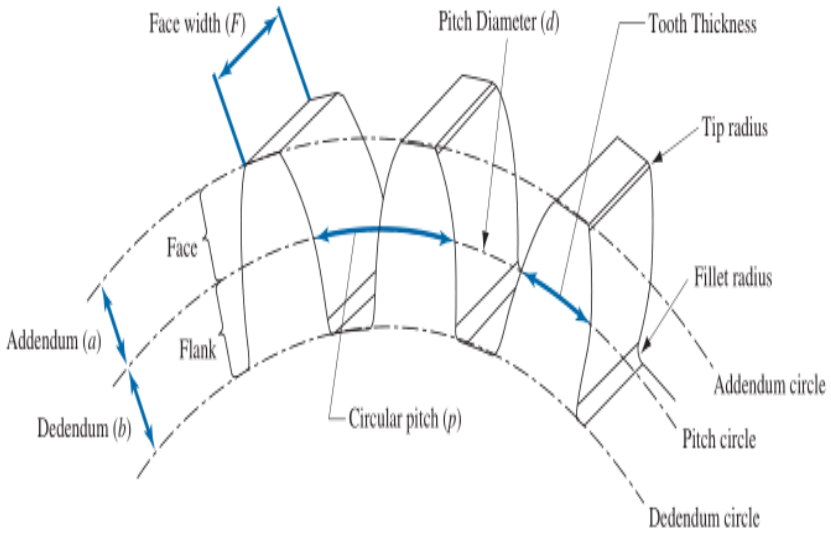
$$x = \left[\left(\frac{3}{2\pi}\right)\theta + 2\right]\cos\theta$$

$$y = \left[\left(\frac{3}{2\pi}\right)\theta + 2\right]\sin\theta$$

Gears



Gear types.



Spur gear tooth features.

$$p = \frac{\pi d}{N}$$

The diametral pitch, P_d : or simply pitch, actually refers to the tooth size and has become a standard for tooth size specifications. Formally, the diametral pitch is the number of teeth per inch of pitch diameter.

$$P_d = \frac{N}{d}$$

The module, m: The module is a relative measure of tooth size.It is defined as the ratio of pitch diameter to the number of teeth in a gear.

$$m = \frac{d}{N}$$

Substituting equations

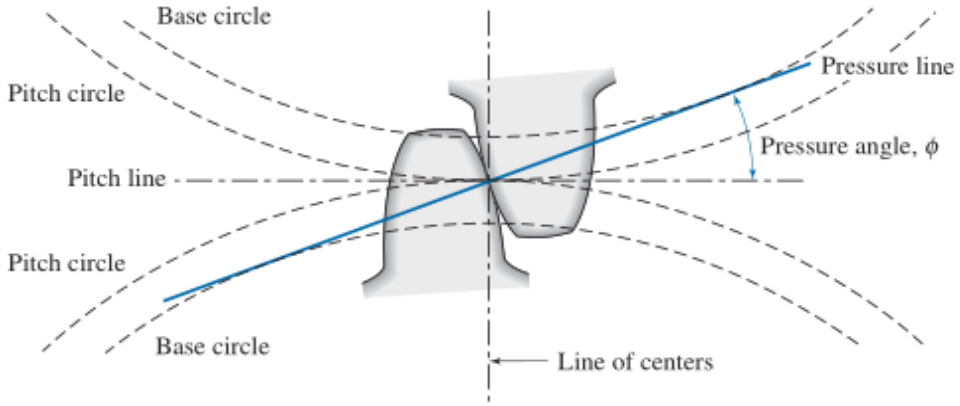
$$p = \frac{\pi d}{N} = \frac{\pi}{P_d} = \pi m$$

The base circle of a gear is the circle from which the curved shape of the gear tooth is constructed.

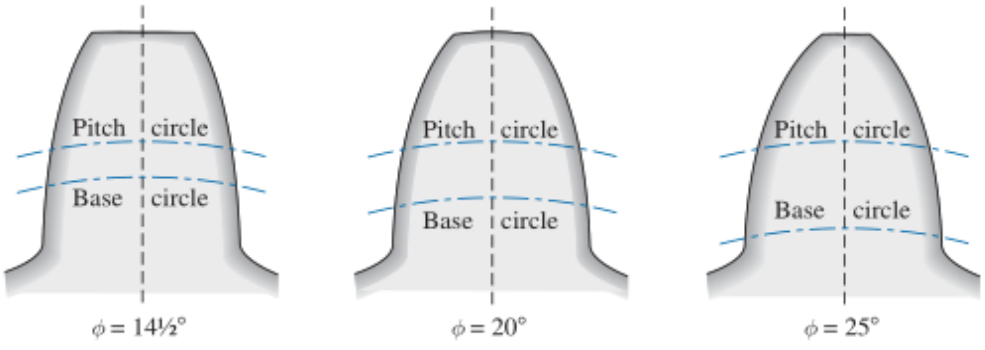
The base diameter, d_b , is the diameter of the circle from which the gear tooth profile is derived.

The face width, F, is the length of the gear tooth parallel with the shaft axis.

The pressure angle, φ, is the angle between a line tangent to both pitch circles of mating gears and a line perpendicular to the surfaces of the teeth at the contact point. The line tangent to the pitch circles is termed the pitch line. The line perpendicular to the surfaces of the teeth at the contact point is termed the pressure line or line of contact.

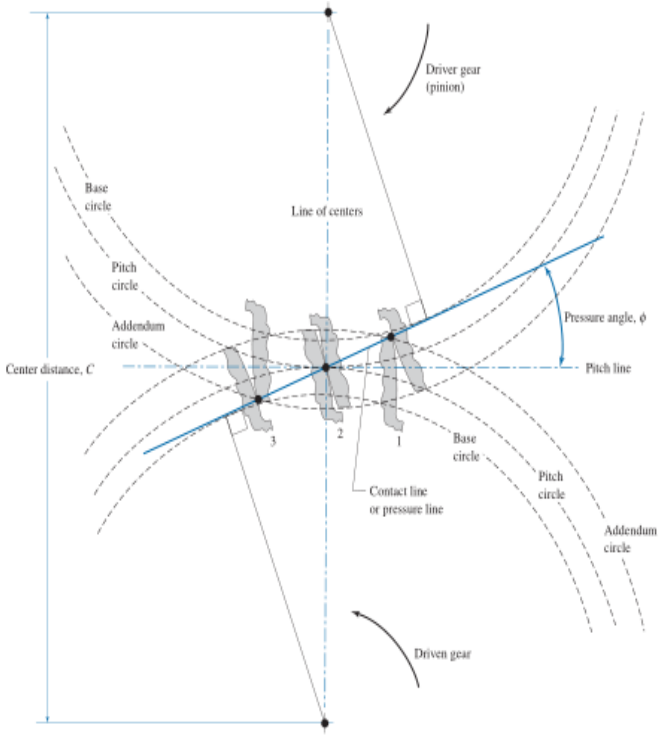


Pressure angle.



Pressure angle influence on tooth shapes.

STANDARD GEARS



Gear mating process.

pressure angle, pitch diameter, and base circle diamet
expressed by

$$d_b = d\cos\phi$$

TABLE 10.3 AGMA Full-Depth Gear Tooth Specifications

Tooth Feature	Coarse Pitch ($P_d < 20$)	Fine Pitch ($P_d \geq 20$)
Pressure angle, ϕ	$14\frac{1}{2}°$ or $20°$ or $25°$	$20°$
Addendum, a	$\frac{1.000}{P_d}$	$\frac{1.000}{P_d}$
Dedendum, b	$\frac{1.250}{P_d}$	$0.002 + \frac{1.2}{P_d}$
Working depth, h_k	$\frac{2.000}{P_d}$	$\frac{2.000}{P_d}$
Whole depth, h_t	$\frac{2.250}{P_d}$	$0.002 + \frac{2.200}{P_d}$
Circular tooth thickness, t	$\frac{1.571}{P_d}$	$\frac{1.571}{P_d}$
Fillet radius, r_f	$\frac{0.300}{P_d}$	not standardized
Min. clearance, c	$\frac{0.250}{P_d}$	$0.002 + \frac{0.200}{P_d}$
Clearance (ground tooth), c	$\frac{0.350}{P_d}$	$0.002 + \frac{0.350}{P_d}$
Min top land width	$\frac{0.250}{P_d}$	not standardized
AGMA standard	201.02	207.04
Face width	$\frac{12}{P_d}$	$\frac{12}{P_d}$

SPUR GEAR KINEMATICS

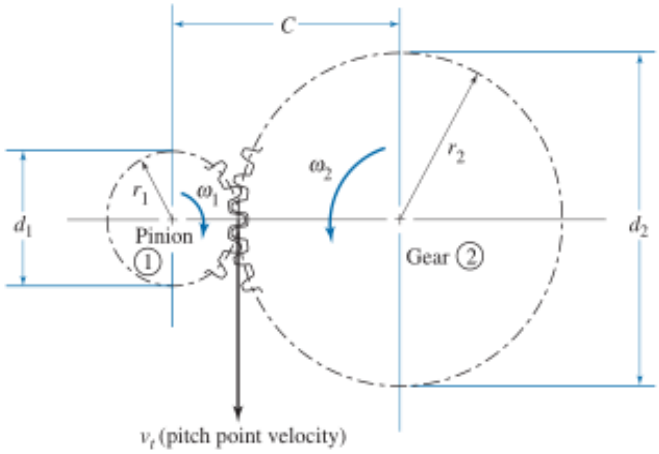
A basic function of gears is to provide a constant velocity ratio between their respective shafts.

A pair of gears that has a constant velocity ratio means that the driven gear maintains a uniform speed as long as the driver gear rotates at a constant speed.

This condition led to the development of the involute tooth profile. A pair of mating spur gears is shown in Figure

Formally, the velocity ratio,V_R, is defined as the angular speed of the driver gear (gear 1) divided by the angular speed of the driven gear (gear 2).

$$VR = \frac{\omega_{driver}}{\omega_{driven}} = \frac{\omega_1}{\omega_2}$$



Kinematics of meshing gears.

The pitch line velocity, v_t, is defined as the magnitude of the velocity of the pitch point of the two mating gears.

This velocity is also illustrated in Figure

$$v_t = r_1\omega_1 = r_2\omega_2$$

Combining equations gives the following relation:

$$\frac{\omega_1}{\omega_2} = \frac{r_1}{r_2} = VR$$

Introducing the pitch diameters,

$$\frac{d_2}{d_1} = \frac{(2r_2)}{(2r_1)} = \frac{r_2}{r_1} = VR$$

and introducing the diametral pitch and number of teeth,

$$\frac{d_2}{d_1} = \frac{\frac{N_2}{P_d}}{\frac{N_1}{P_d}} = VR$$

Because the diametral pitch of the two gears must be identical for the teeth to mate, P_d can be eliminated from the previous equation, yielding

$$\frac{d_2}{d_1} = \frac{N_2}{N_1} = VR$$

Collecting all the preceding relationships yields a comprehensive expression for the velocity ratio.

$$VR = \frac{\omega_1}{\omega_2} = \frac{r_2}{r_1} = \frac{d_2}{d_1} = \frac{N_2}{N_1}$$

An algebraic sign convention designates the relative direction of gear rotations. In the typical external gearset, the shaft centers are on opposite sides of the common tangent to the pitch circles, which dictates that the gears rotate in opposite directions. To signify this fact, the velocity ratio is given a negative value.

For internal gears, as shown in Figure the shaft centers are on the same side of the common tangent to the pitch circles. This dictates that the gears rotate in the same direction. Thus, the velocity ratio is given a positive value.

RELATIONSHIPS OF GEARS IN MESH

.As two gears mesh, the smaller gear is commonly termed the pinion and the larger is referred to as the bull gear or simply the gear.

Recall that in order for two gears to mate,they must have the same diametral pitch and pressure angle

Center Distance

The center distance, C, is defined as the center-to-center distance between two mating gears. This is also the distance between the shafts that are carrying the gears. For the common configuration of external gears the distance can be written as

$$C_{\text{external gears}} = r_1 + r_2 = \frac{(d_1 + d_2)}{2}$$

because

$$d = \frac{N}{P_d}$$

$$C_{\text{external gears}} = \frac{(N_1 + N_2)}{2P_d}$$

For internal gears the center distance is the difference in the pitch radii and can be written as

$$C_{\text{internal gears}} = r_2 - r_1 = \frac{(d_2 - d_1)}{2} = \frac{(N_2 - N_1)}{2P_d}$$

Contact Ratio

The *contact ratio*, m_p, is the average number of teeth that are in contact at any instant. Obviously, the contact ratio must exceed 1 because contact between gears must not be lost. In practice, contact ratios should be greater than 1.2. Robust designs have contact ratios of 1.4 or 1.5. To illustrate the principle, a contact ratio of 1.2 indicates that one pair of teeth is always in contact and a second pair of teeth is in contact 20 percent of the time.

Greater contact ratio values result in smoother action because another gear tooth shares the load for a longer duration during the engaging/disengaging process. In addition, with more teeth sharing the load, greater power may be transmitted. However, the most direct manner in which the contact ratio can be increased is to use larger gears. This is in direct contrast to most design goals of compactness.

Numerically, contact ratio can be expressed as the length of the path of contact divided by the base pitch, p_b. The base pitch, in turn, is defined as the distance between corresponding points of adjacent teeth, measured on the base circle. The base pitch can be computed with the following:

$$p_b = \frac{\pi d_1 \cos\phi}{N_1} = \frac{\pi d_2 \cos\phi}{N_2}$$

Of course, the path of the contact point, of involute gear teeth, is a straight line (Section 10.4). The length of this contact path, Z, is derived by the intersections of the respective addendum circles and the contact line. This length is derived [Ref. 9] as

$$Z = \sqrt{(r_2 + a_2)^2 - (r_2\cos\phi)^2} - r_2\sin\phi + \sqrt{(r_1 + a_1)^2 - (r_1\cos\phi)^2} - r_1\sin\phi$$

Thus, an expression for the contact ratio, in terms of the gear tooth geometry, can be given as

$$m_p = \frac{Z}{p_b}$$

Interference

As mentioned earlier, the most serious drawback with using involute gear tooth profiles is the possibility of interference. Gear teeth have involute profiles between the base circle and the addendum circle. When a gear with few teeth and small pressure angles is constructed, the dedendum circle is considerably smaller than the base circle of the involute. Therefore, the tooth between the base circle and the dedendum is not an involute. If the mating gear tooth were to contact this portion of the tooth, the fundamental condition for constant velocity ratio would be violated. This condition is termed *interference* and, as it occurs, the teeth can exhibit drastic noise, vibration, and wear.

Interference is induced as designers attempt to make gear assemblies compact by using too few teeth on the gears. Interference commonly occurs when a small gear mates with a much larger one. A relationship that can be used to determine the necessary number of teeth in the gear to avoid interference has been derived

The relationship is given as a function of

the number of teeth in the mating pinion, along with the pressure angle and addendum size.

$$N_2 < \frac{\{N_1^2 \sin^2\phi - 4k^2\}}{4k - 2N_1 \sin^2\phi}$$

where k is defined from the addendum relation

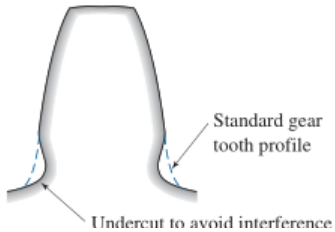
$$a = \frac{k}{P_d}$$

Gear Teeth Combinations to Ensure No Interference

$\phi = 14°$		$\phi = 20°$		$\phi = 25°$	
Number of Pinion Teeth	Maximum Number of Gear Teeth	Number of Pinion Teeth	Maximum Number of Gear Teeth	Number of Pinion Teeth	Maximum Number of Gear Teeth
< 23	Interference	< 13	Interference	< 9	Interference
23	26	13	16	9	13
24	32	14	26	10	32
25	40	15	45	11	249
26	51	16	101	12	∞
27	67	17	1309		
28	92	18	∞		
29	133				
30	219				
31	496				
32	∞				

Undercutting

Interference can also be avoided by removing the material on the gear tooth between the base circle and dedendum circle.



Backlash

As stated in Section 10.3, backlash is the amount that the width of a tooth space exceeds the thickness of a gear tooth, measured on the pitch circle. In more practical terms, it is the amount that a gear can turn without its mating gear turning. Although backlash may seem undesirable, some backlash is necessary to provide for lubrication on the gear teeth. Gears that run continuously in one direction can actually have considerable backlash. Gears that frequently start/stop or reverse direction should have closely controlled backlash.

A nominal value of backlash is designed into a gear tooth profile. The amount of backlash determines the thickness of a gear tooth because backlash is a measure of the tooth thickness to the tooth space. Recommended values of backlash are specified by AGMA [Standard 2002-B88]. Although these values are somewhat conservative, general power-transmitting gears have recommended backlash values of

$$\frac{0.05}{P_d} < B_{\text{recommended}} < \frac{0.1}{P_d}$$

For commercially available stock gears, backlash values are considerably higher to allow for greater flexibility in applications. The backlash values of these gears are typically

$$\frac{0.3}{P_d} < B_{\text{stock gears}} < \frac{0.5}{P_d}$$

Therefore, great care must be taken when specifying stock gears for applications with reversing directions or frequent start/stop sequences.

Backlash values are strongly influenced by any variation in the center distance of the gears. Of course, in any production environment, the center distance of two gears varies. However, a deviation in the nominal center distance can be purposely specified by the designer to adjust the backlash to a desired range. The backlash variation ΔB that will be encountered with a variation in the center distance ΔC can be approximated by the following relationship:

$$\Delta B \approx 2(\Delta C)\tan\phi$$

Operating Pressure Angle

As mentioned in preceding sections, the pressure angle defines the line of action of the force onto the gear teeth. The designated pressure angle is cut or formed into the gear tooth and affects the actual shape of the tooth

It should be mentioned that as the center distance of the mating gears deviates from the nominal value, the actual pressure angle during operation differs from the designated value. In other words, two 20° gears may actually have a greater pressure angle during operation by increasing the center distance from the nominal value. The relationship that can be used to determine the amount of variance is derived [Ref. 9] as

$$\cos\phi_{operating} = \left\{ \frac{C_{nominal}}{C_{operating}} \right\} \cos\phi_{nominal}$$

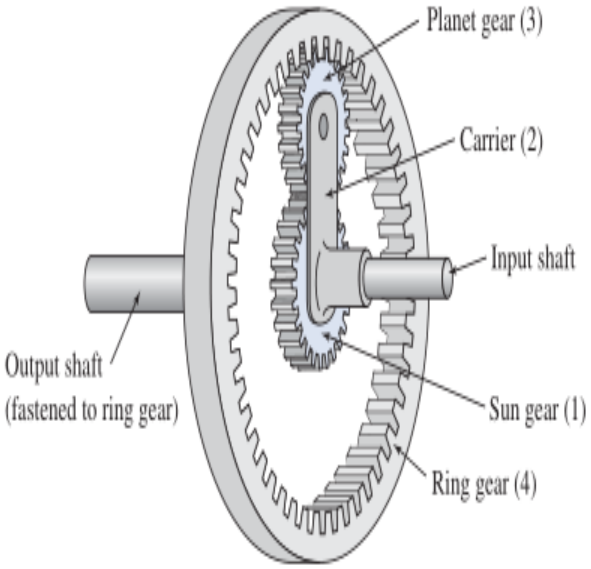
Applications that require precise calculation of the actual force being transmitted should use this operating pressure angle. This reflects the actual performance of the gear forces.

PLANETARY GEAR TRAINS

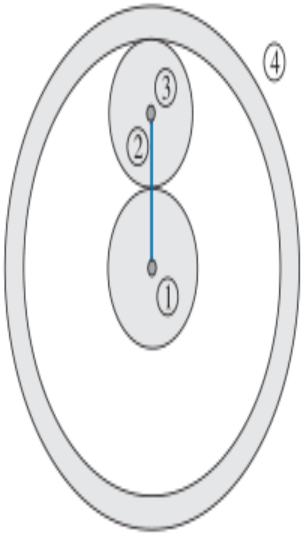
The gear trains presented in preceding sections all had gear centers attached to fixed bodies. With planetary gear trains, this restriction is removed. In these trains, a link that holds the center of the gears is allowed to move. A planetary gear train, which is also called an *epicyclic train*, is shown in Figure

Planetary trains can be used to achieve large speed reductions in a more compact space than a conventional gear train. However, a greater benefit is the ability to readily alter the train value. Because all links are capable of moving, one can alter the train value by holding different gears or carriers. In practice, switching the fixed link is accomplished with brake or clutch mechanisms, thus releasing one link and fixing another. For this reason, planetary gear trains are very common in automotive transmissions.

Because the motion can resemble the planets rotating about the sun in our solar system, the term *planetary gear train* was applied to this system. Expanding on this comparison, the center gear is called the *sun*. Gears that revolve around the sun are called *planets*. A carrier holds the planet gears in orbit around the sun. Finally, the train is commonly encased in the internal gear termed the *ring gear*. These gears are labeled in Figure



(a)



(b)

A planetary gear train.

Planetary Gear Analysis by Superposition

The motion of a planetary gear train is not always as intuitive as fixed-center trains. As gears and carriers rotate, the motion can appear rather complex. To analyze the motion of a planetary gear train, the method of superposition can be used to "step through" the gear movements.

The method of superposition consists of the following:

Step One

The first step is to relax the constraint on the fixed link and temporarily assume that the carrier is locked. Turn the previously fixed gear one revolution and calculate the effect on the entire train.

Step Two

The second step is to free all constraints and record the movement of rotating each link one revolution in the opposite direction of the rotation in step one. As this motion is combined with the motion in the first step, the superimposed motion of the fixed gear equals zero.

Step Three

The motion of all links is determined by combining the rotations from the first two steps. Finally, velocities are proportional to the rotational movements.

Stated in general terms, this method seems complex. However, it is rather straightforward. The method is best illustrated with an example problem.

Planetary Gear Analysis by Equation

Along with the tabular method, the motion of a planetary gear train can also be analyzed through an equation that is derived from the relative angular velocities. To develop the formula method, the motion of the mating gears is examined relative to the carrier. Thus, kinematic inversion is used to view the train is as if the carrier were fixed. One gear on the end of the train is designated the first gear. The gear on the opposite end of the train is designated the last gear.

The train is comprised of meshing gear pairs consisting of driver and driven gears. The first gear is designated as a driver gear and the last gear is a driven gear. The intermediate gears are appropriately identified depending on whether they drive or are driven. In computing the ratio for each pair, the ratio is negative for mating external gears and positive for gears having an internal mesh.

Shifting focus to absolute velocities, the first gear has an angular velocity designated ω_F and the last gear has an angular velocity designated ω_L. The carrier has an angular velocity $\omega_{carrier}$. The relationship between the angular velocities and number of teeth in the train is given as follows.

$$\frac{\omega_{L/carrier}}{\omega_{F/carrier}} = \pm \frac{\text{product of number of teeth on driver gears}}{\text{product of number of teeth on driven gears}} = \frac{\omega_L - \omega_{carrier}}{\omega_F - \omega_{carrier}}$$

GEAR TRAINS

A gear train is a series of mating gearsets. Gear trains are commonly used to achieve large speed reductions. Many mechanical power sources, such as engines, turbines, and electric motors, operate efficiently at high speeds (1800–10,000 rpm). Many uses for this power, such as garage door openers, automotive drive wheels, and ceiling fans, require low speeds (10–100 rpm) for operation. Therefore, a desire to achieve large-velocity reductions is common, and the use of gear trains is very common.

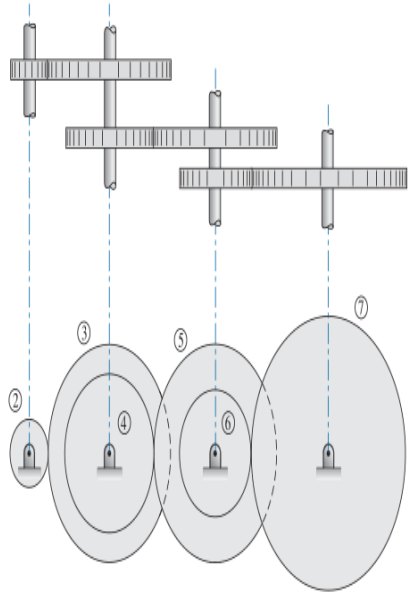
For example, it may be desired to reduce the speed of a shaft from 1800 rpm to 10 rpm. Thus, a velocity reduction of 180:1 is required. If this reduction were attempted with one gearset, equation would reveal that the driven gear would be 180 times larger than the drive gear. Obviously, the driven gear would be tremendously large, heavy, and expensive.

A second, more logical option is to reduce the speed in steps, through a series of gear pairs. This strategy cascades the rotational velocities toward the desired output velocity. This is exactly the logic behind gear trains.

When multiple gear pairs are used in a series, the overall velocity ratio is termed a *train value*—*TV*. The train value is defined as the input velocity to the gear train divided by the output velocity from the train. This is consistent with the definition of a velocity ratio. A train value is the product of the velocity ratio of the individual mating gear pairs that comprise the train. In equation form, it is stated as

$$TV = \frac{\omega_{in}}{\omega_{out}} = (VR_1)(VR_2)(VR_3)$$

The algebraic sign resulting from the multiplication of individual velocity ratios determines the relative rotational direction of input and output shafts. Positive values reveal that the input and output shafts rotate in the same direction, and negative values indicate opposite rotation.



Gear train

Belts and Chains

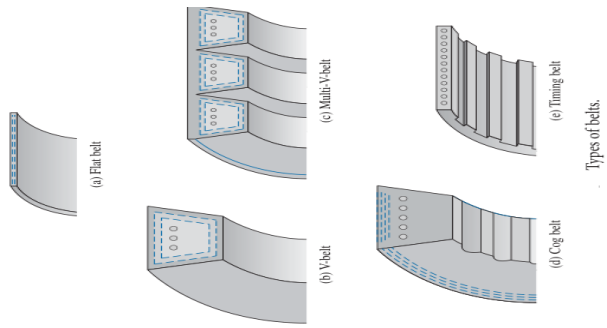
INTRODUCTION

The primary function of a belt or chain drive is identical to that of a gear drive. However, the use of gears becomes impractical when the distance between the shafts is large. Both belt and chain drives offer the flexibility of efficient operation at large and small center distances.

Many belt shapes are commercially available and are listed here.

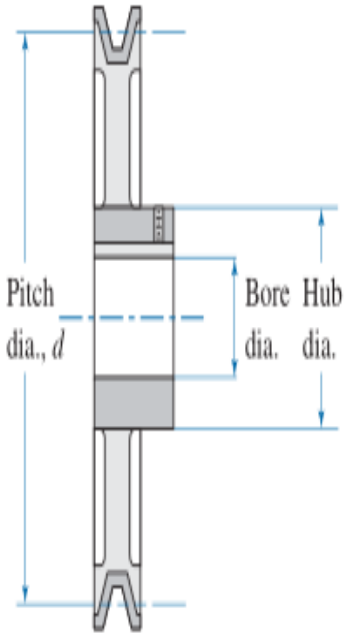
1. A flat belt is shown in Figure. This belt is the simplest type but is typically limited to low-torque applications because the driving force is restricted to pure friction between the belt and the pulley.
2. A V-belt is shown in Figure. This is the most widely used type of belt, particularly in automotive and industrial machines. The V shape causes the belt to wedge tightly into the pulley, increasing friction and allowing higher operating torque.
3. A multi-V-belt is shown in Figure. This belt design is identical to several V-belts placed side by side but is integrally connected. It is used to increase the amount of power transferred.

4. A cog belt is shown in Figure 11.1d. This belt design is similar to a V-belt but has grooves formed on the inner surface. This feature increases belt flexibility, allowing the belt to turn smaller radii. Thus, it can be used on smaller pulleys, reducing the size of the drive.
5. A timing belt is shown in Figure 11.1e. This belt design has gear-like teeth that engage with mating teeth on the pulleys. This arrangement combines the flexibility of a belt with the positive grip of a gear drive. This belt is widely used in applications where relative positioning of the respective shafts is desired

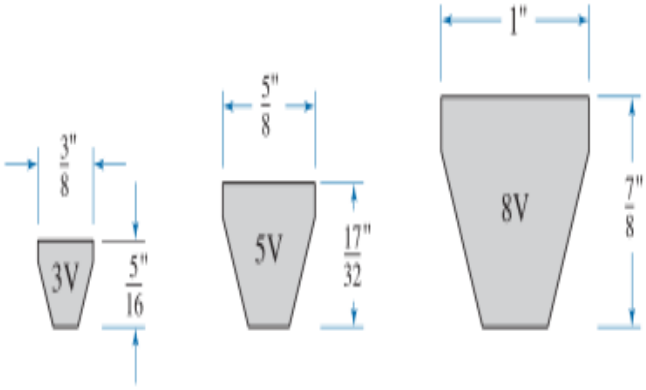


Types of belts.

Pulleys, more appropriately referred to as sheaves, are the wheels that are connected to the shafts and carry the belt



Single-groove V-belt sheave.



Industrial narrow-section V-belts.

BELT DRIVE GEOMETRY

.

As stated, the pitch diameter, d, of the sheave is measured to the point in the groove where the center of the belt sits. This is slightly smaller than the outside diameter of the sheave. Note that the diameters shown for the sheaves in Figures are the pitch diameters.

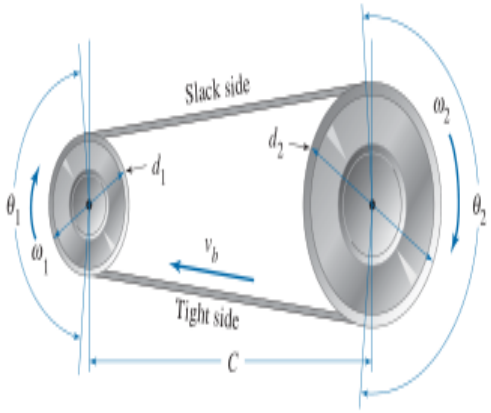
The center distance,C, is the distance between the center of the driver and driven sheaves. Of course, this is also the distance between the two shafts coupled by the belt drive

Normal center distances for V-belts should be in the following range:

$$d_2 < C < 3(d_1 + d_2)$$

The belt length, L, is the total length of the belt.Specifically,the outside length is usually specified

$$L = 2C + \frac{\pi}{2}(d_2 + d_1) + \frac{(d_2 - d_1)^2}{4C}$$



Belt drive geometry.

and

$$C = \frac{B + \sqrt{B^2 - 32(d_2 - d_1)^2}}{16}$$

where

$$B = 4L - 2\pi(d_2 + d_1)$$

The *angle of contact*, θ, is a measure of the angular engagement of the belt on each sheave. It can be computed for each sheave as follows:

$$\theta_1 = 180^\circ - 2\sin^{-1}\left\{\frac{d_2 - d_1}{2C}\right\} \qquad (11.4)$$

$$\theta_2 = 180^\circ + 2\sin^{-1}\left\{\frac{d_2 - d_1}{2C}\right\} \qquad (11.5)$$

BELT DRIVE KINEMATICS

In a manner identical to gear drives, the *velocity ratio*, VR, is defined as the angular speed of the driver sheave (sheave 1) divided by the angular speed of the driven sheave (sheave 2).

$$VR = \frac{\omega_{driver}}{\omega_{driven}} = \frac{\omega_1}{\omega_2}$$

Because a ratio is valid regardless of units, the velocity ratio can be defined in terms of revolutions per minute, radians

The power ratings for commercially available belts, as shown in Figure 11.4, are for drives with sheaves of the same size. Thus, the "rated" angle of contact is 180°. For smaller

Reduced Power Capability with Contact Angle

Angle of Contact, θ	180°	160°	140°	120°	100°	80°
Actual Capability (% of rated power)	100	95	89	82	74	63

per time, or any other convenient set of rotational velocity units. Using the same logic as the derivation of equation yields the following equation:

$$\frac{\omega_1}{\omega_2} = \frac{r_2}{r_1} = VR$$

Introducing the pitch diameters gives

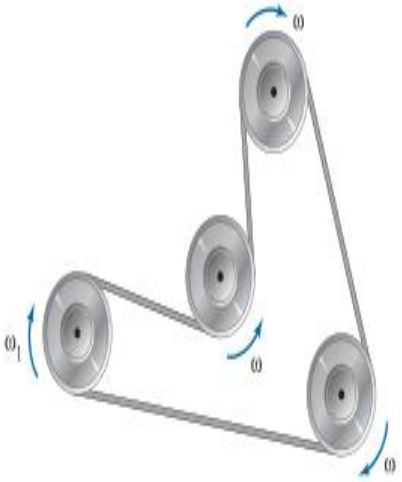
$$\frac{d_2}{d_1} = \frac{2r_2}{2r_1} = \frac{r_2}{r_1} = VR$$

Thus, a comprehensive definition of a velocity ratio is given as

$$VR = \frac{\omega_1}{\omega_2} = \frac{r_2}{r_1} = \frac{d_2}{d_1}$$



(a) Cross drive



(b) Serpentine drive

Alternate forms of belt drives.

rotational velocities of the sheaves and their pitch radii using equation

$$v_b = r_1\omega_2 = \frac{d_1}{2}\omega_1 = r_2\omega = \frac{d_2}{2}\omega_2$$

Note that, the angular velocity in this equation must be specified in radians per unit time.

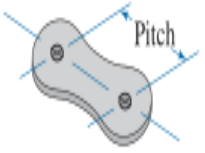
CHAINS

As with belts, chain drives are used to transmit rotational motion and torque from one shaft to another,smoothly,quietly,and inexpensively.Chain drives provide the flexibility of a belt drive with the positive engagement feature of a gear drive.Therefore,chain drives are well suited for applications with large distances between the respective shafts, slow speed, and high torque.

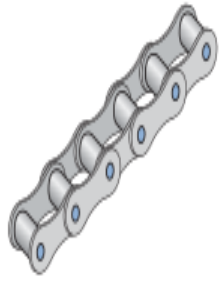
Types of Chains

Chains are made from a series of interconnected links.Many types of chain designs are commercially available and are listed here.

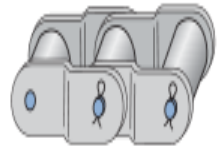
1. A roller chain is shown in Figure. This is the most common type of chain used for power transmission.
2. 2. A multiple-strand roller chain is shown in Figure



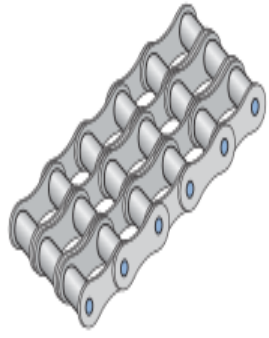
Chain pitch.



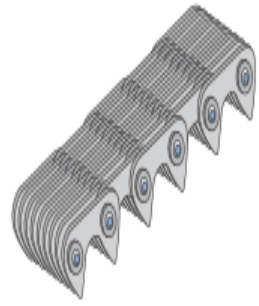
(a) Roller chain



(c) Offset sidebar roller chain



(b) Multiple-strand roller chain



(d) Silent chain

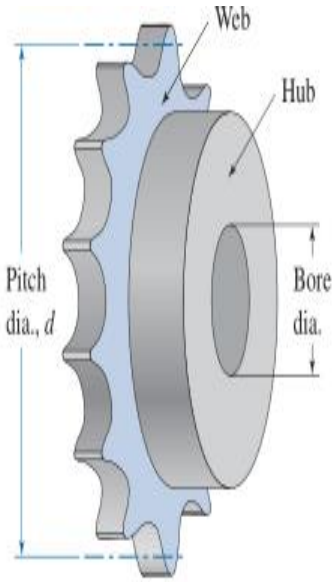
Types of chains.

Sprockets

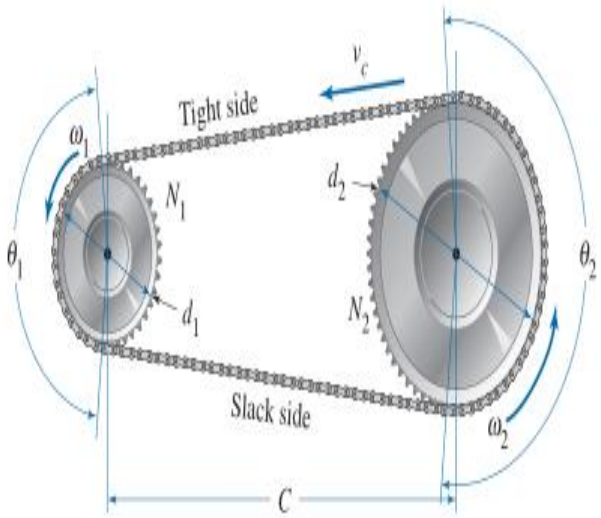
Sprockets are the toothed wheels that connect to the shaft and mate with the chain

CHAIN DRIVE GEOMETRY

As stated, the pitch diameter, d, of the sprocket is measured to the point on the teeth where the center of the chain rides



Roller chain sprocket.



Chain drive geometry.

The pitch diameter of a sprocket with N teeth for a chain with a pitch of p is determined by

$$d = \frac{p}{\sin(180°/N)}$$

The center distance,C, is the distance between the center of the driver and driven sprockets

$$30p < C < 50p$$

The chain length, L, expressed in number of links, or pitches, can be computed as

$$L = \frac{2C}{p} + \frac{(N_2 + N_1)}{2} + \left\{ \frac{p(N_2 - N_1)^2}{4\pi^2 C} \right\}$$

The center distance for a given chain length can be computed as

$$C = \frac{p}{4}\left[L - \frac{(N_2 + N_1)}{2} + \sqrt{\left\{ L - \frac{(N_2 + N_1)}{2} \right\}^2 - \frac{8(N_2 - N_1)^2}{4\pi^2}} \right]$$

The *angle of contact*, θ, is a measure of the angular engagement of the chain on each sprocket. It can be computed as

$$\theta_1 = 180^\circ - 2\sin^{-1}\left\{ \frac{p(N_2 - N_1)}{2C} \right\}$$

$$\theta_2 = 180^\circ + 2\sin^{-1}\left\{ \frac{p(N_2 - N_1)}{2C} \right\}$$

CHAIN DRIVE KINEMATICS

Once again, the velocity ratio, VR, is defined as the angular speed of the driver sprocket (sprocket 1) divided by the angular speed of the driven sprocket (sprocket 2).

$$VR = \frac{\omega_{\text{driver}}}{\omega_{\text{driven}}} = \frac{\omega_1}{\omega_2} = \frac{d_2}{d_1} = \frac{N_2}{N_1}$$

Similar to belts, the linear velocity of the chain, or *chain speed*, is defined as

$$v_c = \frac{d_1}{2}\omega_1 = \frac{d_2}{2}\omega_2$$